Homework 3.

(1) Let $f, g : \mathbb{R} \to \mathbb{R}$ be uniformly continuous functions. Assume that both f and g are bounded. Prove that the product fg is uniformly continuous.

(2) A function $f : \mathbb{R} \to \mathbb{R}$ is periodic, if there exists a $c \in \mathbb{R}$ such that $f(x+c) = f(x)$ for all $x \in \mathbb{R}$. Prove that a continuous periodic function on $\mathbb{R}$ is bounded and uniformly continuous.

(3) Problem 28-1.

(4) Problem 28-3.

(5) If f is differentiable at $a \in (0, 1)$, prove that

$$f'(a) = \lim_{n\to\infty} n \left[f(a + \frac{1}{n}) - f(a) \right].$$

Show by means of an example that the existence of the above limit does not imply the existence of $f'(a)$.