Homework 2, Additional Problems.

(1) Define $f : \mathbb{R} \to \mathbb{R}$ as follows:

$$f(x) = \begin{cases} \frac{1}{q} & \text{if } x = \frac{p}{q} \text{ in smallest terms with } q > 0, p, q \in \mathbb{Z} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Prove that f is continuous at every irrational number and discontinuous at every rational number.

(2) Let X be a non-empty set with the discrete metric. prove that X is complete.