Homework 11, due April 18

**1.** Evaluate $\int_\gamma f(z)\,dz$ when

**a.** $f(z) = |z|^2$ and $\gamma$ is the line segment $[-1+i, 1+i]$ starting at $-1+i$ and ending at $1+i$.

**b.** $f(z) = \text{Re } z$ and $\gamma(t) = t + it^2$ for $0 \le t \le 1$

**c.** $f(z) = \frac{1}{z}$ and $\gamma$ is the join of the line segments $[1-i, 1+i]$, $[1+i, -1+i]$,and $[-1+i, -1-i]$, starting at $1-i$ and traversing the curve once (see figure 1).

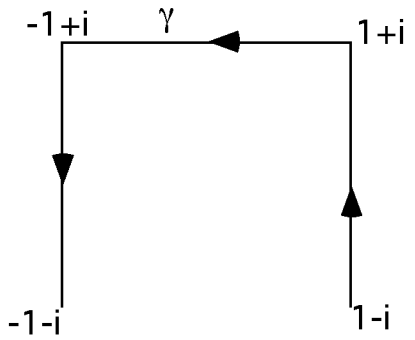


FIGURE 1. $\gamma$

**2.** Compute

$$\int_0^{2\pi} e^{\cos t}[\cos(\sin t + t)]\,dt$$

and

$$\int_0^{2\pi} e^{\cos t}[\sin(\sin t + t)]\,dt$$

by computing $\int_\gamma e^z\,dz$ by using Theorem 2.6 of chapter 2 and where $\gamma(t) = e^{it}$ with $0 \le t \le 2\pi$.

**3.** Let $\gamma(t) = e^{it}$ with $0 \le t \le 2\pi$. Compute (with justification, but without expressing the integral as an integral from 0 to $2\pi$) the following integrals:

**a.** $\int_\gamma \frac{1}{z-2}\,dz$

**b.** $\int_\gamma \frac{\sin z}{z}\,dz$.

**4.** Let $\alpha \in \mathbb{C}$ with $|\alpha| \neq 1$. Compute

$$\int_0^{2\pi} \frac{d\theta}{1 - 2\alpha\cos\theta + \alpha^2}$$

by integrating $(z-\alpha)^{-1}(z-\frac{1}{\alpha})^{-1}$ over the unit circle.