

Homework 11.

- (1) Let $F, G : [a, b] \rightarrow \mathbb{R}$ be absolutely continuous functions. Prove that the product FG is absolutely continuous.
- (2) Let $F : [0, 1] \rightarrow \mathbb{R}$ such that $F'(x)$ exists a.e. and satisfies $F' \in L^1([0, 1])$. Assume F is continuous at 0 and absolutely continuous on $[\epsilon, 1]$ for all $\epsilon > 0$. Prove that F is absolutely continuous on $[0, 1]$ and thus of bounded variation on $[0, 1]$.
- (3) Let $a > b > 0$ and define $F(0) = 0$, $F(x) = x^a \sin \frac{1}{x^b}$ for $0 < x \leq 1$. Prove that F is of bounded variation on $[0, 1]$ (Hint: Use Problem 2 to prove that F is absolutely continuous).