Homework 10, due April 11

1. Prove that
$$f(z) = \sqrt{(|xy|)}$$
satisfies the Cauchy-Riemann equations at $z = 0$, but is not differentiable at $z = 0$.

2. Find all solutions of:

a. $e^z = -i$

b. $\sin(z) = 0$

3. Find power series expansions around $z = 0$ and indicate the radius of convergence for

a. $f(z) = \frac{1}{1+z^3}$

b. $f(z) = \frac{1}{(z+1)(z+2)}$

4. Let $G \subset \mathbb{C}$ be open and let $f \in H(G)$. Let $G^* = \{z : \bar{z} \in G\}$ and define $f^*(z) = \overline{f(\bar{z})}$ for all $z \in G^*$. Prove that $f^* \in H(G^*)$ and express $f^*(z)'$ in terms of f'.

5. Describe the range γ^* and the way in which it is traversed for the following curves γ:

a. $\gamma(t) = 1 + ie^{it} \quad (0 \leq t \leq \pi)$.

b. $\gamma(t) = e^{it} \quad (-2\pi \leq t \leq \pi)$.

c.
$$\gamma(t) = \begin{cases} e^{it}, & 0 \leq t \leq \pi \\ e^{-it}, & \pi \leq t \leq 2\pi. \end{cases}$$