Homework 10

(1) Evaluate $\int_\gamma f(z)\,dz$ using a branch of the log, where $f(z) = \frac{1}{z}$ and γ is the join of the line segments $[1-i, 1+i]$, $[1+i, -1+i]$,and $[-1+i, -1-i]$, starting at $1-i$ and traversing the curve once (see figure 1).

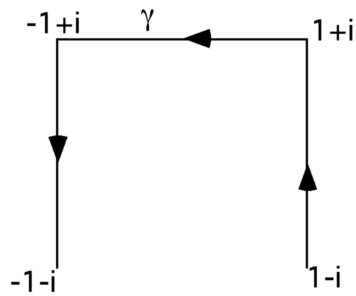

FIGURE 1. γ

(2) Compute

$$\int_0^{2\pi} e^{\cos t}[\cos(\sin t + t)]\,dt$$

and

$$\int_0^{2\pi} e^{\cos t}[\sin(\sin t + t)]\,dt$$

by computing $\int_\gamma e^z\,dz$ using $\int_\gamma f(z)\,dz = \int_0^{2\pi} f(\gamma(t))\gamma'(t)\,dt$, where $\gamma(t) = e^{it}$ with $0 \le t \le 2\pi$.

(3) Evaluate (without parametrizing) $\int_\gamma \frac{1}{1+z^2}\,dz$ for

a. $\gamma(t) = 1 + e^{it}$ $(0 \le t \le 2\pi)$.

b. $\gamma(t) = -i + e^{it}$ $(0 \le t \le 2\pi)$.

c. $\gamma(t) = 2e^{it}$ $(0 \le t \le 2\pi)$.

d. $\gamma(t) = 3i + 3e^{it}$ $(0 \le t \le 2\pi)$.

(4) Let $\alpha \in \mathbb{C}$ with $|\alpha| \ne 1$.

a. Show

$$\int_0^{2\pi} \frac{d\theta}{1 - 2\alpha\cos\theta + \alpha^2} = \frac{i}{\alpha}\int_\gamma \frac{dz}{(z-\alpha)(z-\frac{1}{\alpha})}$$

where $\gamma(t) = e^{it}$ with $0 \le t \le 2\pi$.

b. Compute the integral in part a (hint: use partial fractions).