

Homework 1.

- (1) Let A and B be sets in the range of a function f . Prove that $f^{-1}(A) \setminus f^{-1}(B) = f^{-1}(A \setminus B)$.
- (2) Prove that $[-1; 1)$ is not compact by using the definition of a compact set (to get credit for the problem, use the definition and not any theorems about compact sets).
- (3) What is an interior point? Prove that $\frac{1}{4}$ is an interior point of $(0; 2]$.
- (4) Let $a_1 = \sqrt{6}$ and $a_{n+1} = \sqrt{6 + a_n}$ for $n \geq 1$.
 - a. Show that $a_n \leq 3$ for all $n \geq 1$.
 - b. Show that $\{a_n\}$ is an increasing sequence.
 - c. Explain why $\{a_n\}$ converges.
 - d. Determine the value of $\lim_{n \rightarrow \infty} a_n$.
- (5) Complete the table below indicating $\text{Int}(E)$ (the interior of E), the set of isolated points of E , and whether E is open, closed, both, or neither. An answer of "open" or "closed" in the next to last column will mean that you think E is "open and not closed" or "closed and not open" respectively. You do not need to show work on this problem.

E	$\text{Int}(E)$	Isol. pts. of E	Open? or Closed?	Compact
$(-1, 1]$				
$(0, \infty) \cap \mathbb{Q}$				
\mathbb{R}				
$[2, \infty)$				

- (6) Suppose p is a limit point for two sets A and B .
 - a. Must p be a limit point for $A \cup B$? Justify your answer.
 - b. Must p be a limit point for $A \cap B$? Justify your answer.