Homework 1.

(1) Prove that $[-1;1)$ is not compact by using the definition of a compact set (to get credit for the problem, use the definition and not any theorems about compact sets).

(2) What is an interior point? Prove that $\frac{1}{4}$ is an interior point of $(0;2]$.

(3) Let $a_1 = \sqrt{6}$ and $a_{n+1} = \sqrt{6 + a_n}$ for $n \geq 1$.
  a. Show that $a_n \leq 3$ for all $n \geq 1$.
  b. Show that $\{a_n\}$ is an increasing sequence.
  c. Explain why $\{a_n\}$ converges.
  d. Determine the value of $\lim_{n\to\infty} a_n$.

(4) Complete the table below indicating $\text{Int}(E)$ (the interior of E), the set of isolated points of E, and whether E is open, closed, both, or neither. An answer of "open" or "closed" in the next to last column will mean that you think E is "open and not closed" or "closed and not open" respectively. You do not need to show work on this problem.

E	$\text{Int}(E)$	Isol. pts. of E	Open? or Closed?	Compact
$(-1,1]$				
$(0,\infty)\cap\mathbb{Q}$				
$\mathbb{R}$				
$[2,\infty)$				

(5) Suppose p is a limit point for two sets A and B.
  a. Must p be a limit point for $A \cup B$? Justify your answer.
  b. Must p be a limit point for $A \cap B$? Justify your answer.