Math 546 Course Outline

Spring 2008

Text : *Contemporary Abstract Algebra*
Houghton Mifflin, ISBN 0-618-41471-6 (5th or 6th edition)
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Web page for this course : http://www.math.sc.edu/˜schep/math546-2008.html
Web page for the textbook : http://www.d.umn.edu/ jgallian/
Office hours : MW 10:30–12:00 (or by walk-in or appointment)
Tests : 2 one hour tests, each counting 25%, 1 Final exam counting 35%
Tentative test dates: February 18, April 16
HW+Quizzes : 15%
Homework assignments : http://www.math.sc.edu/˜schep/homework546-2008.html
Date of Final Exam : Wednesday, May 7 - 5:30 p.m.
Material to be covered : Chapters: 0, 1, 2, 3, 4 , 5, 6, 7 and some parts of chapters 8 and 9 time permitting.
The goal of this course is to provide an introduction to abstract algebra, in particular the theory of groups. The emphasis will be on the definitions, theorems and proofs, but we will check these definitions in examples and homework exercises.
Make-up policy : No make-ups for missed homework or quizzes (lowest 2 or 3 scores will be dropped), make-ups for missed hourly tests or final will only be given if they were missed for legitimate reasons. In this case any effort should be made to contact me as soon as possible and you might need to provide documentation to support your reasons for missing the tests.
Attendance policy : A grade penalty can be invoked, if more than 10% of classes are missed.
Cellphone policy: Please set your phone to vibrate if you want to leave it on. No text messaging allowed during class.
University of South Carolina Honor Code It is the responsibility of every student at the University of South Carolina Columbia to adhere steadfastly to truthfulness and to avoid dishonesty, fraud, or deceit of any type in connection with any academic program. Any student who violates this Honor Code or who knowingly assists another to violate this Honor Code shall be subject to discipline.

From the author's website:

Here are some remarks about how to do algebra problems.
1. Never assume a group is Abelian. Some people begin their argument for Exercise 16 of Chapter 2 by saying "Assume that the group is Abelian." This is incorrect for you have no reason to assume a group is Abelian. Many groups are not Abelian.
2. Never divide group elements. Instead, use cancellation or inverses.
3. Never assume a group is finite when that condition was not stated.
4. After you finish a proof look to see if you have used all the hypotheses. For example, if you were given that the group is Abelian check to see if you used that condition in your argument. If the group is finite check to see where you used finiteness. Occasionally, it may be the case that a given condition is not really needed but was there just to make the problem easier but usually all the given conditions are needed for the you to be able to give a valid proof with what you know at this point in the book.
5. Many exercises in the book involve a parameter n and ask you to prove something. (For example, Exercises 15, 19, and 20 in Chapter 2). You should look at the cases for small values of n such as 2 and 3 to gain insight and look for a pattern. This often tells you how to do the general case but keep in mind that doing specific values for n does not do the general case. The problem must be done for all n, not a few examples. In general, you cannot prove a statement is true by using specific examples.
6. When ask to provide an example to illustrate something, D4 is often a good group to try. For example, Exercises 6 and 16 of Chapter 2.
7. On problems such as Exercise 20 of Chapter 2 or Exercise 14 of Chapter 4 do not just give an answer. Show that your answer is valid. You must give reasons or an explanation of why your answer is correct.
8. In many cases problems can be solved by simply writing out the expressions. For example in Exercise 26 of Chapter 2 write out (ab)2 = a2 b2 as abab = aabb. Exercise 19 in Chapter 2 works the same way. Just write the expression out.
9. When you are asked to prove a statement you must not assume that the statement is true.
10. Many theorems in the book about groups and elements of groups involve divisibility conditions and greatest common divisors of two integers. Divisibility only applies to integers. Infinity is not an integer. Do not talk about an integer dividing infinity or an integer being relatively prime to infinity.
11. Whenever you say "Assume ..." you must have a reason why you may assume what it is you are assuming. For example, if you are given that H

is a subgroup of G you may make the statement: Assume x is an element of H because subgroups are not empty. You cannot say "Assume G is Abelian" without providing some reason why you may assume that G is Abelian. As another example, if you are given that a group is finite and a is an element of the group you may say "Assume —a— = n" because all elements of a finite group have finite order. However, if you do not know that the group is finite you can't assume that an arbitrary element from the group has finite order. Instead, you should take two cases. Case 1: —a— is finite and Case 2: —a— is infinite.

12. When asked for an example of something, use a specific example. For instance, in response to Exercise 6 of Chapter 2 some people say that matrices have the property that a-1ba is not equal to b. But you must actually give the specific matrices since some matrices have the desired property and some do not have the property.

13. In general, you cannot take roots (square roots, cube roots, etc.) in groups. Only integer powers of group elements are permissible.

14. When doing a problem about the order of an element, such as proving that an element and its inverse have the same order, you will usually have to deal with the finite case and infinite case separately. That is, —a— = n is one argument and —a— is infinity is a different case. This is usually true as well when dealing with the order of a group. The cases of a finite group and an infinite group may require different arguments.

15. When an exercise says prove something is true for an integer do not assume the integer is positive. In general, the cases that an integer is positive and an integer is negative require slightly different arguments. Usually, you can use the positive integer case to prove the negative integer case by using the Law of Exponents. To illustrate the technique consider Exercise 19 in Chapter 2. To prove (a-1ba)n = a-1bna for all n , first prove it for positive n by writing out the expression a-1ba n times and canceling all the inner a and a-1 terms. (Alternatively, you could use induction.) Now to prove the statement when n is negative observe that a-1bna = ((a-1ba)-n) -1 and that -n is positive. So, since you have already done the case when the exponent is positive you have (a-1 ba)n = ((a-1ba)-n)-1 = (a-1b-n a)-1. Then using the socks-shoes property you have (a-1b-na)-1 = a-1 bna. Finally, the case that n = 0 follows because any element to the 0th power is the identity by definition.

16. When dealing with an abstract group (that is, one in which the elements and operation are not specified) use e to denote the identity and use multiplication as the operation (that is, ab). If you are told the operation is addition use a + b .

17. If you argue by contradiction, don't end it by saying "a contradiction." You

must indicate what you are contradicting (usually this will be the hypothesis or a theorem).

18. The negation "for all" is "there exist some." For example, in an Abelian group ab = ba for all a and b. So, in a non-Abelian group there exist SOME elements a and b such that ab is not ba. To remember this think of a common statement such as "The team won every game." The negation is "There exist some game the team did not win."

19. In the text it is usually the case that elements of a group are denoted by letters from the beginning of the alphabet a,b, c or end of the alphabet x,y,z. Integers such as exponents and orders of elements or groups are usually denoted with letters from the middle of the alphabet i,j,k,m,n,s,t. For example, let —a— = n. You should use the same conventions.

20. NEVER use "if and only if" arguments when the statement is not an "if and only if" statement. Your argument is likely to be wrong since most statements are not "if and only if" and even when they are most of the time "if and only if" arguments are more difficult to make.

21. When you are given an "if and only if" statement to prove it is highly recommended that you do not use an "if and only if" argument. They are tricky to get correct for beginners. Instead, if you are asked to prove the A is true if and only if B is true. Assume that A is true and use this assumption to prove B is true. Then begin all over by assuming that B is true and use that to prove A is true. So, in the end you will have two independent proofs.

22. Please keep in mind that if you are given condition A and asked to prove condition B, you will start your proof with condition A and the last line of your proof will be condition B. If you use a proof by contradiction you can assume that A is true and that B is false to lead to a contradiction. Be sure to say what you are contradicting.

23. When asked to find the inverse of an element, always check your answer by multiplying the element and its purported inverse to see if you get the identity. For example, to check that (ab)-1 = b-1a-1 all you need do is observe that abb-1a-1 = e.

24. When you are asked to prove an "or" statement such as "Prove condition A or condition B" you begin by assuming one of them is false and use that to prove the other condition is true. It does not matter which of the two conditions you assume to be false. If you assume A is false and are not able to prove B is true, then assume B is false and try to prove that A is true. (If you assumed that condition A is false and proved condition B is true there is no need to then assume that condition B is false and prove condition A is true.) An example of this is given for Part 3 of the Lemma in Chapter 7. Exercise 11 of Chapter 7 is another example. Here we may assume that H is not R (if H = R+ we are done)

and use this assumption to prove that H = R*. Another way to prove an "or" statement is to assume both conditions are false and obtain a contradiction.

25. Whenever you are asked to prove a set A is equal to a set B, proceed by assuming some element x belongs to A and show that x belongs to B. Then assume some element x belongs to B and prove that x belongs to A. For example, Exercise 16 of Chapter 3 says "Prove C(a) = C(a-1)." So, begin by assuming that x belongs to C(a) and use this assumption to prove that x belongs to C(a-1). Then assume that x belongs to C(a-1) and use this assumption to prove that x belongs to C(a).

26. Proving a mapping is "onto" causes confusion among many students. If you wish to prove that some function f from A to B is onto, let b denote any element of B. You must find some x in A such that f(x) = b (think of b as given and x as an unknown). To do this replace f(x) by the actual formula for f(x) and then solve for x in terms of b. You must check to see whether the solution you obtained is in set A. Here is an example. Say you are asked to prove that f(x) = x2 from the positive reals to the positive reals is onto. We let b be any positive real. Then we must solve the equation x2 = b for x. Note that x = square root of b and x is a positive real so we have proved that f is onto. In contrast, if we have the same function from the positive rationals to the positive rationals the function is not onto since the square root of a positive rational need not be a positive rational.

27. When ask to prove two groups are not isomorphic students often show that some specific mapping does not satisfy the definition of isomorphism. This merely proves that specific mapping is not an isomorphism. It does not preclude that some other mapping may be an isomorphism. Instead, one must show that NO mapping satisfies the definition. This can be done by assuming there is some generic isomorphism and using only properties of isomorphisms derive a contradiction. Examples 5 and 6 of Chapter 6 illustrate how this can be done. Notice that no specific mapping was assumed. Usually the easiest way to prove that two groups are not isomorphic is to show that they do not share some group property. For example, the group of nonzero complex numbers under multiplication has an element of order 4 (the square root of -1) but the group of nonzero real numbers do not have an element of order 4. As another example, we see that S4 is not isomorphic to D12 because D12 has an element of order 12 whereas S4 has elements of orders only 1, 2, 3 and 4. Often it is easiest to proceed by checking if the largest order of any element in each of the groups agree. When the orders of the elements in two groups match you can prove they are not isomorphic by showing that they have a different number of elements of some specific order. Exercise 35 of the Supplemental Exercises for Chapters 5-8 is such a case. When comparing the number of elements of some specific order,

elements of order 2 is often a good choice.