Extra problemset 4, MATH 550

(1) Let $u_1 = x + y$, $u_2 = x - y$, and $u_3 = 2z$.
   (a) Is this an orthogonal curvilinear system?
   (b) Solve for x, y, and z in terms of u_1, u_2, and u_3.
   (c) Determine the scale factors h_1, h_2, and h_3.
   (d) Let $f(u_1, u_2, u_3) = u_1 + u_2 + u_3$. Fund ∇f.

(2) Let $x = u_1^2 - u_2^2$, $y = 2u_1u_2$, and $z = u_3$.
   (a) Show this is an orthogonal curvilinear system.
   (b) Determine the scale factors h_1, h_2, and h_3.
   (c) Let $\mathbf{F} = u_3\mathbf{e_1} + u_1\mathbf{e_2} + u_2\mathbf{e_3}$. Fund $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$.