Extra problemset 3, MATH 550

(1) Compute $\iiint_V \nabla \cdot \mathbf{F}\, dV$, where $\mathbf{F} = (x^2 + xy)\mathbf{e_1} + (y^2 + yz) + \mathbf{e_2}(z^2 + xz)\mathbf{e_3}$, and V is the cube centered at the origin and with faces on the planes $x = \pm 1$, $y = \pm 1$, $z = \pm 1$.

(2) Let $\mathbf{F} = xyz\mathbf{e_1} + (y^2 + 1)\mathbf{e_2} + z^3\mathbf{e_3}$ and let S be the surface of the unit cube $0 \leq x, y, z \leq 1$. Evaluate the surface integral $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$ using
   (a) the divergence theorem
   (b) Stokes's theorem
   (c) direct computation

(3) If $\mathbf{F} = xz\mathbf{e_1} - y\mathbf{e_2} + x^2y\mathbf{e_3}$, use Stokes's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the closed curve consisting of the edges of the triangle with vertices at the points $P_1 = (1, 0, 0)$, $P_2 = (0, 0, 1)$, $P_3 = (0, 0, 0)$ transversed from P_1 to P_2 to P_3, and back to P_1.

(4) Evaluate the surface integral $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$, where $\mathbf{F} = y\mathbf{e_1} + (x - 2x^3z)\mathbf{e_2} + xy^3\mathbf{e_3}$ and S is the surface of the hemisphere $x^2 + y^2 + z^2 = 4$ above the xy plane.

(5) Evaluate the surface integral $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$, where $\mathbf{F} = (y - z)\mathbf{e_1} + (-x - z)\mathbf{e_2} + (x + y)\mathbf{e_3}$ and S is the surface that is part of of the paraboloid $z = 9 - x^2 - y^2$ above the xy plane.