# Extra Credit problems, MATH 554/703I

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1. Let $(a_n)$ be a sequence of real numbers such that for some $\alpha \neq 0$ we have

$$\lim_{n\to\infty} \frac{a_n - \alpha}{a_n + \alpha} = 0.$$

   What can you say about $(a_n)$.

2. Let $\alpha, \beta > 0$. Prove that

$$\lim_{n\to\infty} \sqrt[n]{\alpha^n + \beta^n} = \max\{\alpha, \beta\}.$$

3. Assume $a_n \to a$ in $\mathbb{R}$. Put $s_n = \frac{a_1+\cdots+a_n}{n}$. Prove that also $s_n \to a$. Conversely give an example such that $(s_n)$ converges for some divergent sequence $(a_n)$.

4. Let $f : [0, \infty) \to \mathbb{R}$ such that $f(x) = \sqrt{x}$. Prove that $f$ is uniformly continuous.