

Homework 10 additional problems.

1. Let  $a \leq b$  and define  $F(0) = 0$ ,  $F(x) = x^a \sin \frac{1}{x^b}$  for  $0 < x \leq 1$ . Prove that  $F$  is not of bounded variation on  $[0, 1]$ .
2. Let  $F : [a, b] \rightarrow \mathbb{R}$  be a continuous function and assume that one of its derivatives, say  $\overline{D^+}F$ , is everywhere  $\geq 0$  on  $[a, b]$ . Prove that  $F$  is increasing on  $[a, b]$ . Hint: First show this for a continuous  $G$  with  $\overline{D^+}G(x) \geq \epsilon > 0$  for all  $x$ . Then apply this to  $G(x) = F(x) + \epsilon x$ .