1. (#1 of the Extra problems)

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} = \vec{0},$$

so

$$\iint_S \left(\nabla \times \vec{F}\right) \cdot \vec{n} dS = 0.$$

For the integral over the boundary curve we get

$$\int_C \vec{F} \cdot \vec{T} ds = \int_C F_1 dx + F_2 dy = \int_C x dx + y dy$$

$$= \int_0^{2\pi} \cos\theta d(\cos\theta) + \sin\theta d(\sin\theta)$$

$$= \frac{1}{2}\cos^2\theta \Big|_0^{2\pi} + \frac{1}{2}\sin^2\theta \Big|_0^{2\pi} = 0.$$

2. The boundary curve C is traversed counterclockwise w.r.t. the $y\,z$ -axis. By Stokes's theorem we have

$$\iint_S \left(\nabla \times \vec{F}\right) \cdot \vec{n} dS = \oint_C \vec{F} \cdot \vec{T} ds = \oint_C F_2 dy + F_3 dz.$$

$$= \int_C -y^3 dy = -\int_0^{2\pi} \cos^3\theta d(\cos\theta) =$$

$$= -\frac{1}{4}\cos^4\theta \Big|_0^{2\pi} = 0.$$

3. (Extra problem 3) The boundary curve C of S has equation $x^2 + y^2 = 1$ and is oriented counterclockwise (check this in a picture!). Thus

$$\iint_S \left(\nabla \times \vec{F}\right) \cdot \vec{n} dS = \oint_C F_1 dx + F_2 dy = \oint_C x dx - x dy = -area(\text{inside } C) = -\pi.$$

4. (Extra problem 4)Note first that $z = \dfrac{1 - x^2 - y^2}{5}$, so that $\dfrac{\partial z}{\partial x} = -\dfrac{2x}{5}$ and $\dfrac{\partial z}{\partial y} = -\dfrac{2y}{5}$.

Now

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xz & yz & x^2 + y^2 \end{vmatrix} = (2y - y)\vec{i} - (2x - x)\vec{j} + 0\vec{k} = y\vec{i} - x\vec{j},$$

so $\displaystyle\iint_S \left(\nabla \times \vec{F}\right) \cdot \vec{n} dS = \iint_D -\frac{\partial z}{\partial x} \cdot y - \frac{\partial z}{\partial y} \cdot (-x) dx dy = \iint_D \frac{2xy}{5} - \frac{2xy}{5} dx dy = 0.$

Now $\int_C \vec{F}\cdot\vec{T}ds = \int_C F_1dx + F_2dy = \int_C xzdx + yzdy = \int_C 0\,dx + 0dy = 0.$

5. (Extra problem 5). The boundary curve C of S has equation $x^2 + y^2 = 1$ and is oriented counterclockwise (check this in a picture!). Thus

$$\iint_S \left(\nabla \times \vec{F}\right)\cdot\vec{n}dS = \oint_C F_1dx + F_2dy = \oint_C \left(zx + z^2y + x\right)dx + \left(z^3yx + y\right)dy = \oint_C xdx + ydy = 0.$$