Trig Review 1

Main Topic # 1: [Converting Radians to Degrees and Degrees to Radians]

To convert between radians and degrees we see from above that:

$$180^\circ = \pi \text{ Radians}$$

Radians and Degrees Conversion

To convert θ Radians to x°we use the following formula:

$$\frac{180 \cdot \theta}{\pi} = x^\circ$$

To convert $x^\circ$to θ Radians we use the following formula:

$$\frac{\pi \cdot x}{180} = \theta \text{ Radians}$$

Learning Outcome # 1: [Convert Radians and Degrees]

Problem 1. Convert from degrees to radians:

1. 30°

2. 135°

3. -330°

Problem 2. Convert from radians to degrees:

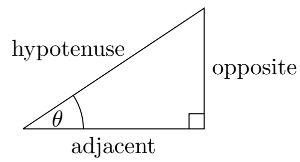
1. $5\pi/6$ radians

2. $\pi/4$ radians

3. $-4\pi/3$ radians

Basic Trig

Basics



$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} \quad \sin(\theta) = \frac{\text{opp}}{\text{hyp}} \quad \tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \quad \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Trig Identities useful in Integration

Pythagorean Identity: $\sin^2(\theta) + \cos^2(\theta) = 1$

Half-Angle Formulas: $\cos^2 x = \frac{1 + \cos(2x)}{2}$ $\qquad$ $\sin^2 x = \frac{1 - \cos(2x)}{2}$

Double-Angle Formulas: $\cos(2x) = \cos^2 x - \sin^2 x$ $\qquad$ $\sin(2x) = 2 \sin x \ \cos x$

Add./Subst. Formulas:

$$\cos(s+t) = \cos s \ \cos t - \sin s \ \sin t$$
$$\sin(s+t) = \sin s \ \cos t + \cos s \ \sin t$$
$$\cos(s-t) = \cos s \ \cos t + \sin s \ \sin t$$
$$\sin(s-t) = \sin s \ \cos t - \cos s \ \sin t$$

Inverse Trig

Basics

$$y = \sin(\theta) \implies \theta = \sin^{-1}(y) \text{ where } -1 \le y \le 1 \text{ and } \frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$y = \cos(\theta) \implies \theta = \cos^{-1}(y) \text{ where } -1 \le y \le 1 \text{ and } 0 \le \theta \le \pi$$

$$y = \tan(\theta) \implies \theta = \tan^{-1}(y) \text{ where } y \in \mathbb{R} \text{ and } \frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$$

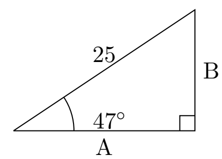
$$y = \cot(\theta) \implies \theta = \cot^{-1}(y) \text{ where } y \in \mathbb{R} \text{ and } 0 \le \theta \le \pi$$

$$y = \sec(\theta) \implies \theta = \sec^{-1}(y) \text{ where } |y| \ge 1 \text{ and } 0 \le \theta \le \pi, \theta \ne \frac{\pi}{2}$$

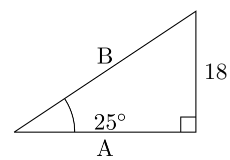
$$y = \csc(\theta) \implies \theta = \csc^{-1}(y) \text{ where } |y| \ge 1 \text{ and } \frac{-\pi}{2} \le \theta \le \frac{\pi}{2}, \theta \ne 0$$

Problem 3. Solve for the missing sides and angles of the following triangles

1. Find the length of sides A, B and the missing angle



2. Find the length of sides A, B and the missing angle



Problem 4. Use sum or difference formula to evaluate the following exactly. There may be more than one way to evaluate each.

(a) $\cos(7\pi/12)$

(b) $\sin(13\pi/12)$

(c) $\sin(\pi/12)$

(d) $\sin(5\pi/12)$

Problem 5. Using the fact that $2\theta = \theta + \theta$, and the sum formulas for sin and cos on the previous page verify (i.e. show algebraically) identities for $\sin(2\theta)$ and $\cos(2\theta)$.

Problem 6. Using the double angle formulas (for example you found them in problem 5) for $\cos(2\theta)$ and define a new variable $u = 2\theta$. Use this new variable to find (i.e. show algebraically) a formula for $\sin(u/2)$.

Problem 7. Use the half-angle formulas to evaluate the following exactly.

(a) $\sin(\pi/12)$

(b) $\cos(11\pi/8)$

Problem 8. Suppose angles A and B are in the first quadrant, and $\sin(A) = \frac{1}{4}$ and $\sin(B) = \frac{12}{13}$.

(a) Find $\cos(A)$ and $\cos(B)$ exactly.

(b) Find $\sin(A + B)$ and $\sin(A - B)$ exactly.

Problem 9. Verify the following identity using formulas you already know.

$$\sin(3\theta) = 3\sin(\theta) - 4\sin^3(\theta)$$

Problem 10. Solve $\cos(x) = -\dfrac{1}{2}$.

Problem 11. Solve $\sin(x) = 0$.

Problem 12. Solve $\cos(x) = \frac{\sqrt{3}}{2}$.

Problem 13. Solve $\sin(2x) = \frac{\sqrt{2}}{2}$ subject to the restriction that $0 \leq x \leq \pi$.

Problem 14. Solve $\cos\left(\frac{1}{4}x\right) = 1$.

Problem 15. Solve $\sin^2(x) - 1 = 0$.

Problem 16. Solve $2\cos^2(x) - 5\cos(x) - 3 = 0$.