

The anticommutativity of the maps in a double complex

SAS pg 2

Stop the map d^V from being mapping $\underline{\text{Ch}}$

we can "fix" this by mapping

$$f_{*q} : C_{*q} \rightarrow C_{*q-1} \text{ by } f_{pq} = (-1)^p d^V_{pq}$$

It is easy to check this works since now

$$(-1)^p d^h_{pq-1} \circ d^V_{pq} = d^h_{pq-1} \circ f_{pq} \stackrel{?}{=} f_{p-1,q} \circ d^h_{pq} = (-1)^{p-1} d^V_{p-1,q} \circ d^h_{pq}$$

which follows since

$$d^V_{p-1,q} \circ d^h_{pq} = - d^h_{pq-1} \circ d^V_{pq}$$

Hint: In R-mod the negatives clearly commute, one should verify that negatives still "commute" in any abelian category Hint: similar to the profinite rings!

This gives us a way to identify double complexes with the category $\underline{\text{Ch}}(\underline{\text{Ch}})$ of chain complexes in the abelian category $\underline{\text{Ch}}$

i.e.

This is why we care about double complexes

next we will see further motivation for the anticommutativity ...

[
Spoiler alert: helps us define the mapping
Core of section 5]

