Long Division

Main Topic # 1: [Long Division with polynomials] The basic concept of worksheet is write rational expressions as as a polynomial plus a simpler rational expression. This idea was seen with numbers when you were younger. More specifically we can find a q called the *quotient* and r the remainder so that

$$\frac{a}{b} = q + \frac{r}{b}$$

so that $r < b$. This is not as tricky as it might seem to the mathematical adverse student. It is only a tricky way of writing that there is a number q such that

$$a = q \cdot b + r$$

that is q is the most amount of times b divides into a an r is the left over, that is why it is less than b if it was greater we could get another factor of b. Now we want to do the same process with polynomials specifically lets look at

$$\underbrace{(x^2 + 2x - 1)}_{\text{dividend}} \div \underbrace{(x - 1)}_{\text{divisor}}$$

We begin very much the same as long division of numbers *but* this time we **only look at** the highest term in each. The *highest term* in $x - 1$ is just x^1 and the highest term in $x^2 + 2x - 1$ is x^2. We even write the set up the same!

$$x - 1\overline{)\ x^2 + 2x - 1}$$

So we first ask the question what can we multiply x (i.e. our highest term in our divisor) to get x^2 (i.e. our highest term in our dividend) well that is of course x or to put it in an equation

$$x \cdot x = x^2$$

We collect this data just as before by writing it above the line

$$\begin{array}{r} x \\ x - 1\overline{)\ x^2 + 2x - 1} \end{array}$$

We then proceed like with numbers and multiply our new factor x by $(x - 1)$ and get

$$x \cdot (x - 1) = x^2 - x$$

and again we must subtract (don't forget to distribute the subtraction sign i.e. $-(x^2 - x) = -x^2 + x$

$$\begin{array}{r} x \\ x - 1\overline{)\ x^2 + 2x - 1} \\ \underline{-x^2 \ \ + x \quad} \end{array}$$

and then performing the subtraction by subtracting like terms we get

$$\begin{array}{r} x \\ x - 1\overline{)\ x^2 + 2x - 1} \\ \underline{-x^2 \ \ + x \quad} \\ 3x - 1 \end{array}$$

Now we repeat the process with the *left over polynomial* in this example it is $3x - 1$. Notice the highest term of $3x - 1$ is $3x$ and we ask what do we need to multiply x (the highest term of $x - 1$) by to get $3x$? We see here it is more simple we need only multiply by 3! That is

$$3 \cdot x = 3x$$

Now to collect this information we **add** the new term 3 above the line

$$\begin{array}{r} x+3 \\ x-1\overline{\smash{)}\; x^2+2x-1} \\ \underline{-x^2\;\;+x} \\ 3x-1 \end{array}$$

And again multiply 3 by $(x-1)$ we get $3x-3$ and when we subtract we get $-(3x-3) = -3x+3$ and we collect this data as follows

$$\begin{array}{r} x+3 \\ x-1\overline{\smash{)}\; x^2+2x-1} \\ \underline{-x^2\;\;+x} \\ 3x-1 \\ \underline{-3x+3} \end{array}$$

again combining like terms we get

$$\begin{array}{r} x+3 \\ x-1\overline{\smash{)}\; x^2+2x-1} \\ \underline{-x^2\;\;+x} \\ 3x-1 \\ \underline{-3x+3} \\ 2 \end{array}$$

and now we see that we are at a degree lower than that of the divisor so we are done! this is our remainder 2! So we have the following:

$$\frac{x^2+2x-1}{x-1} = (x+3) + \frac{2}{x-1}$$

Before you move on make sure you can follow these next examples!

Examples:

$$\begin{array}{r} x+2 \\ x^2-x+1\overline{\smash{)}\; x^3\;\;+x^2+2x-1} \\ \underline{-x^3\;\;+x^2\;\;-x} \\ 2x^2\;\;+x-1 \\ \underline{-2x^2+2x-2} \\ 3x-3 \end{array}$$

$$\text{i.e. } \frac{x^3+x^2+2x-1}{x^2-x+1} = \Big(x+2\Big) + \frac{3x-3}{x^2-x+1}$$

$$\begin{array}{r} \tfrac{3}{4}x+\tfrac{3}{16} \\ 4x^2-x+1\overline{\smash{)}\; 3x^3\qquad\quad +2x\;\;-1} \\ \underline{-3x^3+\tfrac{3}{4}x^2\;\;-\tfrac{3}{4}x} \\ \tfrac{3}{4}x^2\;\;+\tfrac{5}{4}x\;\;-1 \\ \underline{-\tfrac{3}{4}x^2+\tfrac{3}{16}x-\tfrac{3}{16}} \\ \tfrac{23}{16}x-\tfrac{19}{16} \end{array}$$

$$\text{i.e. } \frac{3x^3+2x-1}{4x^2-x+1} = \left(\tfrac{3}{4}x+\tfrac{3}{16}\right) + \frac{\tfrac{23}{16}x-\tfrac{19}{16}}{4x^2-x+1}$$

Problem 1. Write the following as quotient plus remainder and fraction.

1. $\left(3x^4 - 5x^2 + 3\right) \div \left(x + 2\right)$
2. $\left(x^3 + 2x^2 - 3x + 4\right) \div \left(x - 7\right)$
3. $\left(2x^5 + x^4 - 6x + 9\right) \div \left(x^2 - 3x + 1\right)$
4. $\left(x^3 + x^2 + x + 1\right) \div \left(x + 9\right)$
5. $\left(7x^3 - 1\right) \div \left(x + 2\right)$
6. $\left(5x^4 + x^2 - 8x + 2\right) \div \left(x - 4\right)$