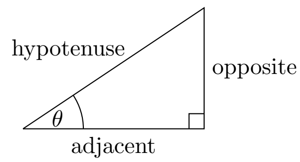
Trig Review 1

Basic Trig

Basics



$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} \quad \sin(\theta) = \frac{\text{opp}}{\text{hyp}} \quad \tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \quad \sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Trig Identities useful in Integration

Pythagorean Identity: $\sin^2(\theta) + \cos^2(\theta) = 1$

Half-Angle Formulas: $\cos^2 x = \dfrac{1+\cos(2x)}{2}$ $\quad \sin^2 x = \dfrac{1-\cos(2x)}{2}$

Double-Angle Formulas: $\cos(2x) = \cos^2 x - \sin^2 x$ $\quad \sin(2x) = 2\sin x\ \cos x$

Add./Subst. Formulas:

$$\cos(s+t) = \cos s\ \cos t - \sin s\ \sin t$$
$$\sin(s+t) = \sin s\ \cos t + \cos s\ \sin t$$
$$\cos(s-t) = \cos s\ \cos t + \sin s\ \sin t$$
$$\sin(s-t) = \sin s\ \cos t - \cos s\ \sin t$$

Problem 1. Using the fact that $2\theta = \theta + \theta$, and the sum formulas for sin and cos on above verify (i.e. show algebraically) identities for $\sin(2\theta)$ and $\cos(2\theta)$.

Problem 2. Using the double angle formulas (for example you found them in problem 5) for $\cos(2\theta)$ and define a new variable $u = 2\theta$. Use this new variable to find (i.e. show algebraically) a formula for $\sin(u/2)$.

Problem 3. Verify the following identity using formulas you already know.

$$\sin(3\theta) = 3\sin(\theta) - 4\sin^3(\theta)$$

Problem 4. Suppose angles A and B are in the first quadrant, and $\sin(A) = \frac{1}{4}$ and $\sin(B) = \frac{12}{13}$.

(a) Find $\cos(A)$ and $\cos(B)$ exactly.

(b) Find $\sin(A + B)$ and $\sin(A - B)$ exactly.