Log Review

Main Topic # 1: [Log is "like" square root]

The main concept for Logs is the concept of the **opposite** in the sense of the inverse of a function. Recall how one calculates the positive branch of the square root:

$$\sqrt{4} = \_\_\_\_$$

That is because

$$(\_\_\_\_)^2 = \_\_\_\_$$

In slight more generality:

$$\sqrt{b} = c$$

means

$$c^2 = b$$

It is the same idea for the Log and an exponential. First the technical definition:

> **The Log**
>
> For $a > 0$ we call the inverse of the function $f(x) = a^x$ **Log base** a and write it as
>
> $$\log_a(x) = f^{-1}(x)$$

Now the definition which mimics the idea of square root above.

> **The Opposite of an Exponential**
>
> For $a > 0$
>
> $$\log_a(b) = c$$
>
> means
>
> $$a^c = b$$
>
> or
>
> $$\log_a(b)$$
>
> is the power of ____ that is ____

That is the Log is the function that says "Gimme that exponent"

Finally, there is a special Log that we call **The Natural Log**:

$$\log_e(x) = \ln(x)$$

Learning Outcome # 1: [Using the meaning of Log to calculate]

Problem 1. Complete the following statements.

(a) If $y = \log_{10}(100)$, then $\_\_\_\_^y = 100$.

(b) $\log_{10}(5.5)$ is the power of ____ that gives ____.

(c) $\log_2$(____) is the power of ____ that gives 500.

(d) If $4^m = n$ then $\log_4(n) =$ ____.

(e) $\log_e(556)$ is the power of ____ that gives ____.

Problem 2. Rewrite the following using exponents instead of logs.

(a) $\log_e(5) \approx 1.609$

(b) $\log_2(1) = 0$

(c) $\log_{100}(A) = B$

Problem 3. Rewrite the following using logs instead of exponents.

(a) $e^{15} \approx 3269017.373$

(b) $10^{-2} = \frac{1}{100}$

(c) $7^t = H$

Problem 4. Evaluate the following without using a calculator:

(a) $3^{\log_3(7)}$

(b) $\log_{11}(11^4)$

(c) $\log_b(\sqrt{b^3})$

Problem 5. Evaluate the following without using a calculator.

(a) $e^{\ln(17)}$

(b) $\ln(e^3)$

(c) $\ln\left(\frac{1}{\sqrt{e}}\right)$

Main Topic # 2: [Solving Equations with Log]

In the last section we talked about exponential functions. Today we want to talk about how to solve equations like:

$$3^x = 7$$

So we need a way to:

"un-do" raising to the x

Just like we did in equations before...

<u>Addition and Subtraction</u>: To solve the equation $x+7=8$ we need to "un-do adding 7" by subtracting 7 from both sides and get:

$$x+7 = 8$$

$$x = 1$$

<u>Multiplication and Division</u>: To solve the equation $3x = 9$ we need to "un-do the multiplcation by 3" by dividing by 3 on both sides of the equation to get :

$$(3)x = 9$$

$$x = 3$$

This is exactly what **the inverse** is for functions. To be more specific when considering the function $f(x) = 3^x$ the inverse has the following property:

$$f^{-1}(f(x)) = x$$

To use this property in the first equation we see:

$$3^x = \quad 9$$

$$x = \log_3(9) = \_\_\_\_$$

The Take-Away

For $a > 0$ we have:

$$a^{\log_a(x)} = x$$

and

$$\log_a(a^x) = x$$

BECAUSE THEY ARE "OPPOSITES"!!!!

Finally, let's look at the graph of the Log, using what we learned about inverses in the previous section:

Sketching Logs

Exponential functions look like:

$0 < a < 1$ $\qquad\qquad$ $1 < a$

So we see that the **Domain** of $\log_a(x)$ is ____

and the **Range** of $\log_a(x)$ is ____

Learning Outcome # 2: [Solving Basic Equations with Log]

Problem 6. Solve for x in the equations below.

(a) $3^x = 29$

(b) $6 = 2(1.03)^x$

(c) $e^{-x} = \frac{1}{2}$

(d) $3^x - 7 = 12$

(e) $3e^{5x} + 2 = 8$

(f) $4^{-7x} - 2 = 10$

(g) $6 \cdot 3^{-2x} = 3^{4x}$

(h) $\left(e^x\right)^4 + 3 = 7$

(i) $2\left(7^x\right)^2 + 3 = 15$

Main Topic # 3: [The Laws of Logs]

Recall the Laws of Exponents

The Laws of Exponents

$$a^n \cdot a^m = a^{m+n}$$
$$(a^n)^m = a^{m \cdot n}$$
$$\frac{a^n}{a^m} = a^{n-m}$$
$$a^0 = 1$$

To understand the Laws of Logs we will first investigate what happens to all of the exponential laws when we apply $\log_a(\_\_)$ to both sides:

$$\log_a\left(a^n \cdot a^m\right) = \log_a\left(a^{m+n}\right) = m + n = \underline{\qquad} + \underline{\qquad}$$

$$\log_a\left(\left(a^n\right)^m\right) = \log_a\left(a^{m \cdot n}\right) = m \cdot n = \underline{\quad} \cdot \underline{\qquad}$$

$$\log_a\left(\frac{a^n}{a^m}\right) = \log_a\left(a^{m+n}\right) = n - m = \underline{\qquad} - \underline{\qquad}$$

$$\underline{\quad} = \log_a\left(a^0\right) = \log_a(1)$$

The awesome thing is the far right hand side has nothing to do with the base, which gives us our Laws of Logs, or as I like to call it:

Rob's Log Laws

The Laws of Logs

$$\log_a(A \cdot B) = \log_a(A) + \log_a(B)$$
$$\log_a\left(A^n\right) = n \cdot \log_a(A)$$
$$\log_a\left(\frac{A}{B}\right) = \log_a(A) - \log_a(B)$$
$$\log_a(1) = 0$$

Learning Outcome # 3: [Identifying and Applying the Laws of Logs]

Problem 7. Match each expression of the left with its equivalent expression on the right for $A, B > 0$.

$\ln(AB)$	$\log_a(A^{3t})$
$\log_a\left(\frac{A}{B}\right)$	$\ln(A) + \ln(B)$
$\log_a(A^2) - \log_a(B)$	1
$t\log_a(A^3)$	$\frac{\ln(A)}{2}$
$\log_a(1)$	$2\log_a\left(\frac{A}{\sqrt{B}}\right)$
$\ln(e)$	0
$\ln(\sqrt{A})$	$\log_a(A) - \log_a(B)$

Problem 8. Rewrite each of the following as the sum/difference of simple logarithms.

(a) $\ln\left(\frac{3x^2}{yz}\right)$ (b) $\log_{10}\left(\frac{a^2 b}{(cd)^3}\right)$ (c) $\log_3\left(\frac{(z-1)^3}{z^{3/2}}\right)$

Problem 9. Rewrite each of the following as a single logarithm.

(a) $\ln(x) + \ln(3) - 2\ln(y)$

(b) $\log_{10}(a) - 2\log_{10}(b) + 3\log_{10}(c) - 4\log_{10}(d)$

(c) $\frac{1}{2}\log_c(x) - \log_c(y) - \log_c(z-1) - \log_c(a)$

Learning Outcome # 4: [Solving Equations using the Laws of Logs]

Problem 10. Solve the following equations:

(a) $3^{x+1} = 9^{2x}$

(b) $6^x = 7^{x-1}$

(c) $3^{2x-1} = 5^x$

Problem 11. Solve the following equations:

(a) $\log_{10}(x-3) = 4$

(b) $\log_2(x) + \log_2(x+2) = \log_2(6x+1)$

(c) $\log_3(x) - \log_3(x-1) = 2$

(d) $2\ln(x) = \ln(x+3) + \ln(x-1)$