

Factoring Polynomials

Common Factors

Using the distributive property we get the first way to factor

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

that is $a \cdot b$ and $a \cdot c$ have a common factor of a .

Example(s):

$$(6x + 3) = 3(2x + 1)$$

$$(2x^2 + 3x) = x(2x + 3)$$

$$a^3b + a^2b^2 - ab^2 = ab(a^2 + ab - b)$$

Factor by Grouping

$$\begin{aligned} a \cdot c + a \cdot d + b \cdot c + b \cdot d &= \underbrace{a \cdot c + a \cdot d}_{\text{group together}} + \underbrace{b \cdot c + b \cdot d}_{\text{group together}} \\ &= a \cdot \underbrace{(c + d)}_{\text{common factor}} + b \cdot \underbrace{(c + d)}_{\text{common factor}} \\ &= (c + d) \cdot (a + b) \end{aligned}$$

That is group together items with common factors... then factor out common factors again!

Example(s):

$$\begin{aligned} 10 + x^2 + 2x + 5x &= \underbrace{x^2 + 2x}_{\text{group together}} + \underbrace{5x + 10}_{\text{group together}} \\ &= \underbrace{x(x + 2)}_{\text{common factor}} + \underbrace{5(x + 2)}_{\text{common factor}} \\ &= (x + 2)(x + 5) \end{aligned}$$

AC Method To factor

$$a + b + c$$

Step 1: Find expressions r and s so that

$$r + s = b$$

$$r \cdot s = a \cdot c$$

Step 2: Rewrite $b = r + s$

$$a + b + c = a + \underbrace{r + s}_{r+s=b} + c$$

Step 3: Factor by Grouping

Example(s):

$$\begin{aligned} x^2 + 5x + 6 &= x^2 + 3x + 2x + 6 \\ &= x(x + 3) + 2(x + 3) = (x + 2)(x + 3) \end{aligned}$$

Common Factors

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Perfect Squares

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$

Problem 1. Factor the following polynomials. If the expression is already factored as much as possible, say so.

1. $y^2 + 2y + 9y + 18$

9. $100x^2 + 180x + 81$

2. $-20 + r^2 + 4r - 5r$

10. $50 - 98r^2$

3. $x^3 + x^2 + x + 1$

11. $16t^2 + 56t + 49$

4. $m^2 + 2nm - 5mn - 10n^2$

12. $k^4 - 16$

5. $m^2 - 25$

13. $25z^2 - 36$

6. $y^2 - 16$

14. $18m^3n + 3m^2n^2 - 6mn^3$

7. $16x^2 - 40x + 25$

15. $6t^2 - 11tu - 7u^2$

8. $81x^4 - 900x^2$

16. $40p - 32r$

17. $36r^2 - 60rs + 25s^2$

22. $2x^2 - 2x - 40$

18. $2z^2 - 7z - 4$

23. $a^4 - 9a^2 + 20$

19. $48y^2z^3 - 28y^3z^4$

24. $1 - x^8$

20. $225k^2 - 36r^2$

25. $5x^6 - 18x^3 + 9$

21. $k^2 - 6k - 16$

26. $-15r^2 + 2r + 24$