Chapter 8
Sections 8.1 & 8.2

Warm-up Problem A. Determine if the ordered pair $(p, q) = \left(-\frac{1}{2}, \frac{3}{4}\right)$ is a solution of each system.

(i) $\begin{cases} 8p + 4q = -1 \\ -\frac{1}{2}p + q = \frac{1}{2} \end{cases}$

$-4+3=-1$ ✓

$\frac{1}{4}+\frac{3}{4}=1$ ✗

(ii) $\begin{cases} -8p + 12q = 13 \\ 4p + 2q = -\frac{1}{2} \end{cases}$

$4+9=13$ ✓

$-2+\frac{3}{2}=\frac{-4+3}{2}=\frac{-1}{2}$ ✓

Warm-up Problem B. Find two numbers whose sum is -2 and whose difference is 35.

$x+y=-2 \Rightarrow y=-2-x \rightsquigarrow y=\frac{-35}{2}$

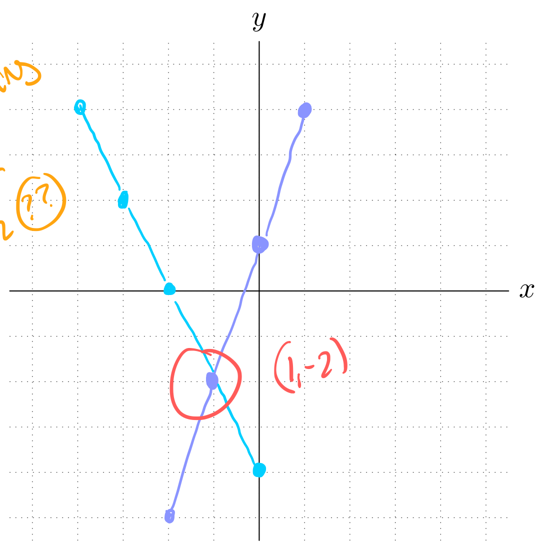
$x-y=35 \Rightarrow x+2+x=35 \Rightarrow 2x=33$

$x=\frac{33}{2}$

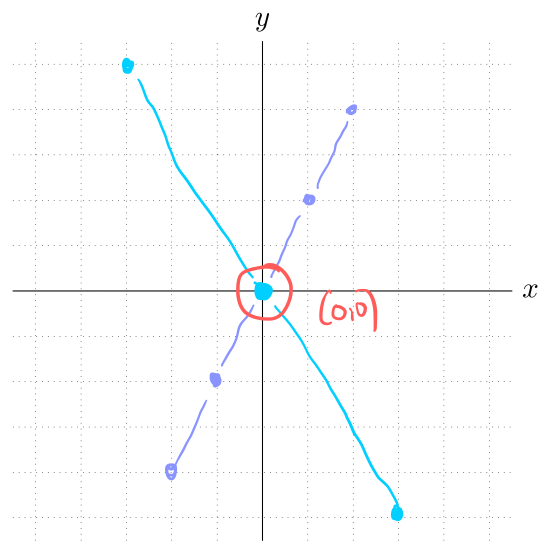
Problem 1. Solve each system by graphing. Check that your solution satisfies *both* equations.

I feel like graphing is better suited for section 7.2 ??

1. $\begin{cases} -3x + y = 1 \\ 2x + y = -4 \end{cases}$



2. $\begin{cases} 2x - y = 0 \\ 5x + 3y = 0 \end{cases}$



Problem 2. Use the method of substitution to solve the following systems.

(a) $\begin{cases} 3x + 7y = 16 \\ y = 2x - 5 \end{cases}$

$3x+14x-35=16$

$17x=51$

$x=\frac{51}{17}=3$, $y=\frac{102}{17}-5=1$

$(3,1)$

(b) $\begin{cases} 2x - 5y = 11 \\ 3x = 2y \end{cases}$

$2x-\frac{15}{2}x=1$

$-\frac{11}{2}x=1$

$x=\frac{-2}{11}$ $y=\frac{-3}{11}$

$\left(\frac{-2}{11}, \frac{-3}{11}\right)$

Problem 3. Use the method of elimination to solve the following systems.

(a) $\begin{cases} 2(3x + 4y = 3) \\ +\ 9x - 8y = 4 \end{cases}$

$15x = 10$

$x = \frac{2}{3}$

$y = \frac{3-2}{4} = \frac{1}{4}$

$\left(\frac{2}{3}, \frac{1}{4}\right)$

(b) $\begin{cases} 4x - 5y = -14 \\ -2(2x + 3y = 4) \end{cases}$

$-11y = -18$

$y = \frac{18}{11}$

$\left(\frac{14}{11}, \frac{-5}{11}\right)$

$x = \frac{\frac{-18(3)}{11} + 4}{2} = \frac{\frac{-10}{11}}{2} = \frac{-5}{11}$

Problem 4. Solve the systems of linear equations.

- Identify which method you use to solve each system (elimination or substitution). Can you explain why you choose that method?
- If the system has one solution, write the solution as an ordered pair (x, y).

(a) $\begin{cases} x + 2y = 5 \\ 2x - y = 4 \end{cases}$

$y = 2x - 4$

$x + 4x - 8 = 5$

$5x = 13$

$x = \frac{13}{5},\ y = \frac{26}{5} - 8 = \frac{14}{5}$

$\left(\frac{13}{5}, \frac{14}{5}\right)$

(c) $\begin{cases} \frac{1}{4}x + \frac{1}{2}y = 12 \\ \frac{1}{6}x + \frac{1}{3}y = 8 \end{cases}$

infinite soln

(b) $\begin{cases} x - 2y = 10 \\ -\frac{1}{2}x + y = -5 \end{cases}$

infinite soln

(d) $\begin{cases} 0.2x + 0.5y = 6 \\ 0.4x + y = 9 \end{cases}$

$y = 9 - 0.4x$

$(15, 3)$

$-0.2x + 9 = 6$

$x = \frac{30}{2} = 15,\ y = 9 - 0.4(15) = 3$

Problem 5. Brent has \$32.00 in nickels and quarters. He has 44 more quarters than nickels. Determine how many coins of each type Brent has.

$5n + 25q = 3200$

$n + 44 = q$

$5n + 25(n+44) = 3200$

$30n + (25 \cdot 44) = 3200$

$n = \frac{3200 - (25 \cdot 44)}{30} = 70$

$q = 114$

Problem 6. The Garden Center ordered 6 ounces of marigold seed and 8 ounces of carnation seed paying \$214.54. They later ordered another 12 ounces of marigold seed and 18 ounces of carnation seed, paying \$464.28. Find the price per ounce for each type of seed.

$C = 29.89$

$m = -4.10$

$6 \cdot m + 8c = 214.54$

$12m + 18c = 464.28$

$[18 - 3(8)]c = 464.28 - 3(214.54) \Rightarrow -6C = -179.34 \Rightarrow C = 29.89$

Problem 7. A taxi charges a flat rate plus a certain charge per mile. A trip of 4 miles costs \$2.05, while a trip of 8 miles costs \$2.85. Find the flat rate and the charge per mile.

x = charge per mile

y = flat rate

$4 \cdot x + y = 2.05$

$8x + y = 2.85$

$y = 1.25$

$4x = 2.85 - 2.05 \Rightarrow x = 0.2$

Problem 8. Solve the system by substitution. Because there are three unknowns to solve for, your solution will be an ordered triple (x, y, z).

$$\begin{cases} 2x + 3y + 4z = 3 \\ \quad\quad\; 8y - 2z = 26 \\ \quad\quad\quad\quad\;\; 3z = -3 \end{cases}$$

$(-1, 3, -1)$

$8y - 2(-1) = 26$

$y = \frac{24}{8} = 3$

$2x + 9 - 4 = 3$

$2x = -2 \quad x = -1$

Problem 9. Without solving the systems, determine the number of solutions each system has. Justify your answer.

(a) $\begin{cases} x - 2y = 5 \\ 2x - 4y = 10 \end{cases}$ (b) $\begin{cases} 8x + 4y = 0 \\ 4x - 2y = 2 \end{cases}$ (c) $\begin{cases} 2x - y = -1 \\ -2x + y = 7 \end{cases}$

infinite

Skalar multiple

* same line *

no sol'n

parallel lines

no sol'n

parallel lines

Problem 10. You take a test in which there are 200 points possible. The test consists of True/False questions worth 2 points each, multiple choice questions worth 5 points each, and essay questions worth 10 points each. There are 14 more multiple choice questions than True/False questions. There are 4 times as many multiple choice questions on the exam as there are essay questions. How many questions of each type are on the exam?

$\to 2T + 5(14+T) + 10(3.5 + .25T) = 200$

$2T + 5m + 10e = 200$ $\quad 9.5T = 95$ $\quad T = 10$

$-T + m = 14 \to m = 14 + T \Rightarrow m = 24$

$m - 4e = 0$ $\quad e = 6$

$m = 4e \Rightarrow e = \frac{1}{4}m = 3.5 + \frac{1}{4}T$

Problem 11. A radiator holds 10 liters. Suppose you have a bottle of pure antifreeze and a bottle 10% antifreeze mixture. How much of each ~~must~~ do you need to make enough of a 20% mixture to fill the radiator?

$x + y \cdot (.1) = .2 \Rightarrow x + (0.1)(10 - x) = 0.2$

$x + y = 10$

$y = 10 - x$

$(1 - 0.1)x = 0.2 - 1$

$$\boxed{x = \frac{0.2 - 1}{1 - 0.1} = 0.\overline{88} \qquad y = 10 - 0.\overline{88} = 9.\overline{11}}$$

Additional Problems

EP 1. Solve the systems of linear equations below.

(a) $\begin{cases} 4x - 6y = -19 \\ 3x - 4y = -14 \end{cases}$

$3x = -80$

$x = \frac{-80}{3}$

$y = \frac{-40+14}{4} = \frac{-33}{2}$

(b) $\begin{cases} x - y = 3 \\ 2x + 2y = 22 \end{cases}$

$y = x - 3$

$2x + 2(x-3) = 22$

$4x - 6 = 22$

$x = \frac{28}{4} = 7 \quad y = 7 - 3 = 4$

EP 2. The length of a rectangular room is 5 feet more than the width. The perimeter of the room is 58 feet. Find the dimensions of the room.

$2(5+x) + 2x = 58$

$10 + 4x = 58$

width → $x = 12$

length → 17

EP 3. Hermione wants to purchase a mix of candy from the Honeydukes Express. After adjusting for exchange rates, she decides to make a mix of Bertie Bott's Every Flavour Beans worth \$12 per kg, and Chocolate frogs worth \$15 per kg to get 120 kg of a mixture worth \$13 per kg. How many kg of each should she buy?

$\frac{12 \cdot B + 15 \cdot F}{120} = 13$

$B + F = 120$

$\Rightarrow$

$12B + 15(120 - B) = 120 \cdot 13$

$-3B + 1800 = 1560$

$-3B = -240$

$B = 80 \quad F = 40$

EP 4. Four friends, Elise, Tom, Garrett and Mary, decide to grab a bite to eat at Runza before class. They each eat the following:

Elise : 1 Original Runza, 1 Medium Fry, 1 Dr. Pepper for a total of 1090 calories

Tom : 1 Cheese Runza, 1 Medium Fry, 1 Dr. Pepper for a total of 1140 calories.

Garrett : 2 Original Runzas, 1 Dr. Pepper for a total of 1250 calories.

Mary : 1 Original Runza, 1 Dr. Pepper for a total of 720 calories.

Let R be the number of calories in one Original Runza, let C be the number of calories in one Cheese Runza, let F be the number of calories in one Medium Fry and let D be the number of calories in one Dr. Pepper.

(a) For each friend, write an equation that gives the number of calories consumed in terms of the variables R, C, F and D.

Elise: $O + F + D = 1090$

Tom: $C + F + D = 1140$

Garrett: $2O + D = 1250$

Mary: $O + D = 720$

(b) Using the equations you wrote for Mary and Garrett, determine how many calories are in one Dr. Pepper, i.e. find the value of D.

$\emptyset + F + D = 1090$

$C + F + D = 1140$

$2\emptyset + D = 1250$

$\emptyset + D = 720$

$\emptyset = 720 - D$

$2(720-D) + D = 1250$

$1440 - 2D + D = 1250 \Rightarrow \boxed{D = 190}$

(c) Using the value you found above for D and an equation from part (a), find the number of calories in one Original Runza, i.e. find the value of R.

$\emptyset = 720 - 190 = 530$

(d) Using the values you found above for D and R, and an equation from part (a), find the number of calories in one Medium Fry, i.e. find the value of F.

$530 + F + 190 = 1090$

$F = 370$

(e) Using the values you found above for D, R and F, and an equation from part (a), find the number of calories in one Cheese Runza, i.e. find the value of C.

$C + 370 + 190 = 1140$, $C = 580$

(f) Fill in the following values:

An Original Runza contains 530 calories.

A Cheese Runza contains 580 calories.

A Medium Fry contains 370 calories.

A Dr. Pepper contains 190 calories.

EP 5. Adam and Natasha row their canoe 28 miles downstream in 2 hours. After a picnic, they row their canoe back upstream. After 3 hours of rowing upstream, they only travel 12 miles. Assuming that Adam and Natasha canoe at a constant rate, and that the river's current is constant, find the speed at which Adam and Natasha can row in still water.

$2(r+s) = 28$

$3(r-s) = 12 \Rightarrow s = \frac{3r-12}{3} = 3r-4 = 3.5$

$2r + 6r - 8 = 28$

$8r = 20 \quad r = \frac{20}{8} = 2.5$