Chapter 5
Section 5.4-5.5

Main Topic # 1: [The other Trig functions] There are other Trig functions that will be important, here is a concise list of them

The Other Trig Functions

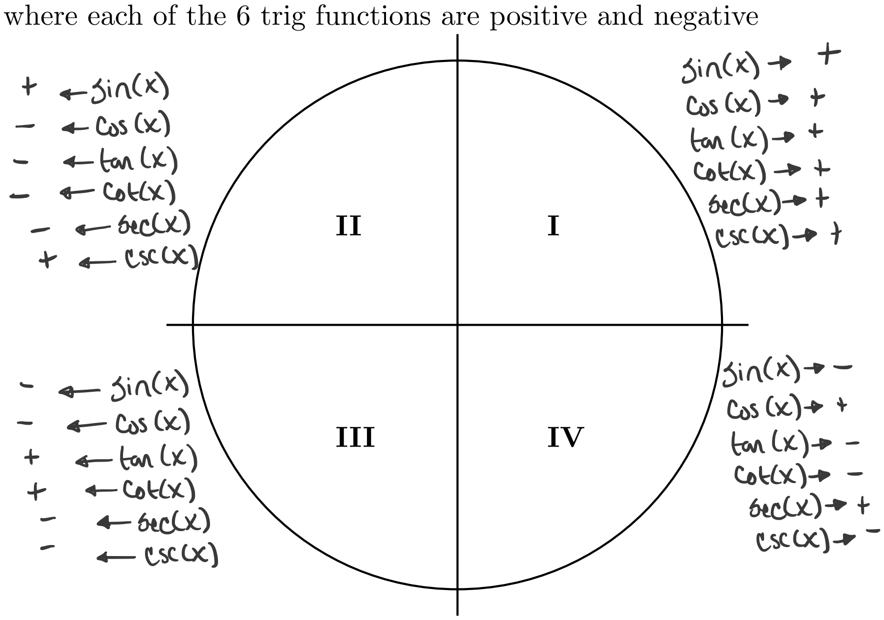
For any angle θ we have the following:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$
$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$
$$\sec(\theta) = \frac{1}{\cos(\theta)}$$
$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

These functions have the trouble that they each have vertical asymptotes.

Where are each positive and negative?

Lets note where each of the 6 trig functions are positive and negative

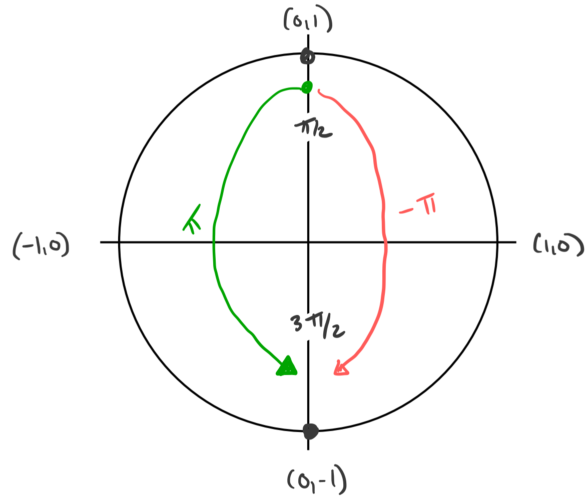


Let's look at $\tan(\theta)$ and $\sec(\theta)$ these both have $\cos(\theta)$ in the denominator, and hence have vertical asymptotes when every $\cos(\theta) = 0$.

Vertical Asymptotes of $\tan(\theta)$ and $\sec(\theta)$

The vertical asymptotes of both $\tan(\theta)$ and $\sec(\theta)$ are when

$$\cos(\theta) = 0$$



$\cos(\theta) = 0$
when

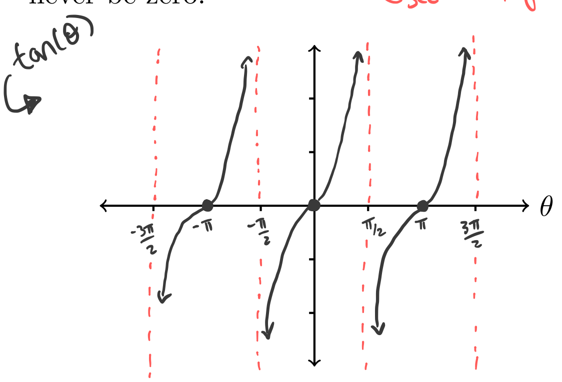
$$\theta = \frac{\pi}{2} + n\pi = \frac{(2n+1)\pi}{2}$$

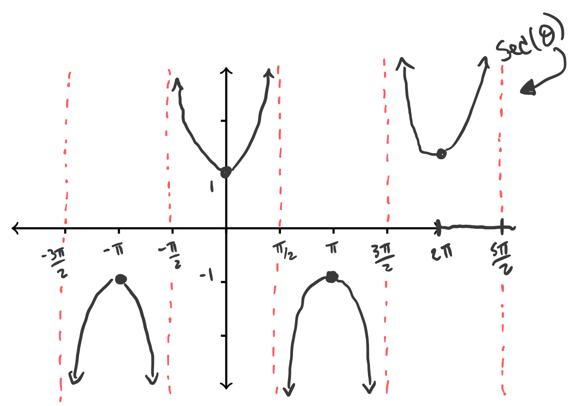
Now we can graph

The Graphs of $\tan(\theta)$ and $\sec(\theta)$

To graph these it will be helpful to know the zeros of the functions. First, $\tan(\theta) = 0$ when $\sin(\theta) = 0$ which is at $\theta = n\pi$ for any integer n. Yet as 1 is in the numerator of $\sec(\theta)$ it can never be zero.

see next page





With these graphs we see a few things:

(i) Domain: $\tan(\theta)$: $\theta \neq \frac{(2n+1)\pi}{2}$ $\qquad$ $\sec(\theta)$: $\theta \neq \frac{(2n+1)\pi}{2} = \frac{\pi}{2} + n\pi$

(ii) Range: $\tan(\theta)$: $(-\infty, \infty)$ $\qquad$ $\sec(\theta)$: $(-\infty, -1] \cup [1, \infty)$

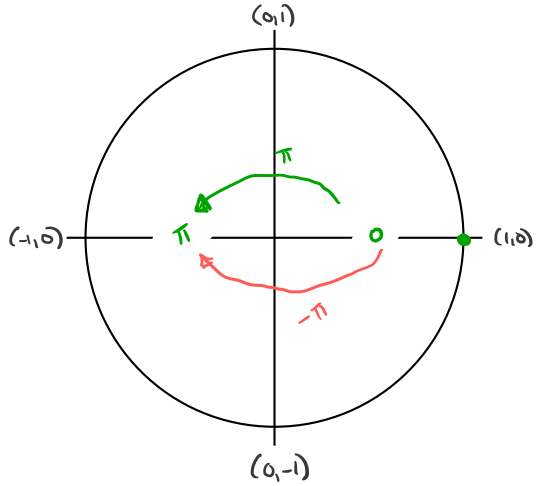
(iii) Period: $\tan(\theta)$: π $\qquad$ $\sec(\theta)$: 2π

Next let's look at $\cot(\theta)$ and $\csc(\theta)$ these both have $\sin(\theta)$ in the denominator, and hence have vertical asymptotes when every $\sin(\theta) = 0$.

Vertical Asymptotes of $\cot(\theta)$ and $\csc(\theta)$

The vertical asymptotes of both $\cot(\theta)$ and $\csc(\theta)$ are when

$$\sin(\theta) = 0$$



$\sin(\theta) = 0$
when
$\theta = 0 + n\pi$
$= n\pi$

Now we can graph

The Graphs of $\cot(\theta)$ and $\csc(\theta)$

To graph these it will be helpful to know the zeros of the functions. First, $\cot(\theta) = 0$ when $\cos(\theta) = 0$ which is at $\theta = \frac{(2n+1)\pi}{2}$ for any integer n. Yet as 1 is in the numerator of $\csc(\theta)$ it can never be zero.

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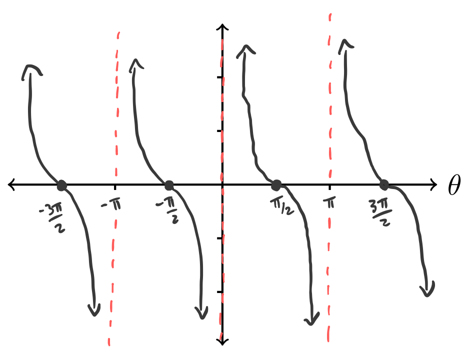
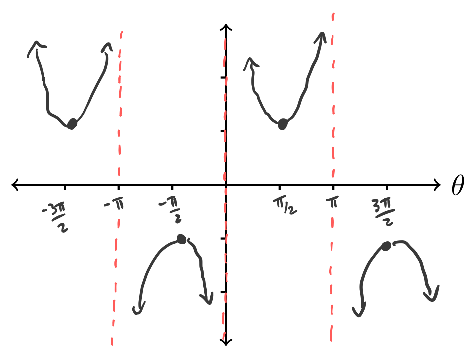
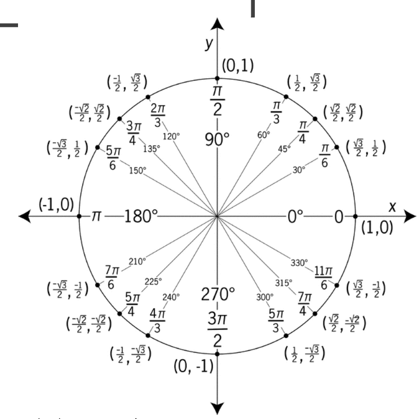
$\cot(\theta)$



$\csc(\theta)$





With these graphs we see a few things:

(i) Domain: $\cot(\theta)$: $\theta \neq n\pi$ $\quad$ $\csc(\theta)$: $\theta \neq n\pi$

(ii) Range: $\cot(\theta)$: $(-\infty, \infty)$ $\quad$ $\csc(\theta)$: $(-\infty, -1] \cup [1, \infty)$

(iii) Period: $\cot(\theta)$: π $\quad$ $\csc(\theta)$: 2π

Learning Outcome # 1: [Using the Unit Circle to Calculate Trig Functions]

Problem 1. Find the value of all 6 trigonometric functions for the following angles:

(a) π

$\sin(\pi) = 0$
$\cos(\pi) = -1$
$\tan(\pi) = 0$
$\cot(\pi) = \text{DNE}$
$\sec(\pi) = -1$
$\csc(\pi) = \text{DNE}$

(b) $\pi/4$

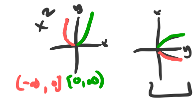
$\sin(\pi) = \frac{\sqrt{2}}{2}$
$\cos(\pi) = \frac{\sqrt{2}}{2}$
$\tan(\pi) = 1$
$\cot(\pi) = 1$
$\sec(\pi) = \frac{2}{\sqrt{2}}$
$\csc(\pi) = \frac{2}{\sqrt{2}}$

(c) $4\pi/3$

$\sin(\pi) = -\frac{\sqrt{3}}{2}$
$\cos(\pi) = -\frac{1}{2}$
$\tan(\pi) = \sqrt{3}$
$\cot(\pi) = \frac{1}{\sqrt{3}}$
$\sec(\pi) =$
$\csc(\pi) = \frac{-2}{\sqrt{3}}$

(d) $-\pi/6$

$\sin(\pi) = -\frac{1}{2}$
$\cos(\pi) = \frac{\sqrt{3}}{2}$
$\tan(\pi) = -\frac{1}{\sqrt{3}}$
$\cot(\pi) = -\sqrt{3}$
$\sec(\pi) = \frac{2}{\sqrt{3}}$
$\csc(\pi) = -2$



Main Topic # 2: [ArcSine and ArcCosine]

We would like to solve equations like: $\sin(x) = \frac{\sqrt{3}}{2}$ $x = \arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

We saw in the section about Logs that this is achieved by the inverse of the function. Yet unlike the situation with Log, we have the problem that the inverse of $\sin(x)$ and $\cos(x)$ are not functions!

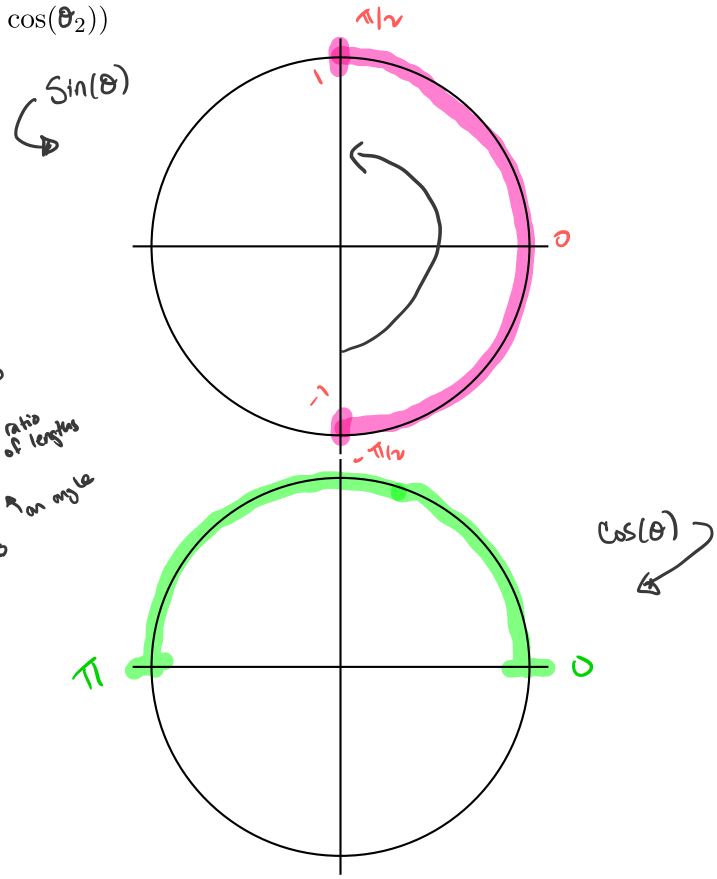
Just like $\sqrt{x}$ to have a useful inverse we will need to find out a useful restriction for the domains of $\sin(x)$ and $\cos(x)$.

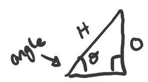
Restricting the Domains of $\sin(x)$ and $\cos(x)$

We want to find a restriction of the domains of $\sin(\theta)$ and $\cos(\theta)$, so that we can solve equations like the one above we will need this restricted domain to satisfy some useful properties

(i) Need a θ so that for every y between -1 and 1 we have that $\sin(\theta) = y$ (or $\cos(\theta) = y$)

(ii) Only want a domain so that when θ_1 and θ_2 are in the domain then $\sin(\theta_1) \neq \sin(\theta_2)$ (or $\cos(\theta_1) \neq \cos(\theta_2)$)





$\sin(\theta) = \frac{O}{H}$ a ratio of lengths

$\arcsin(\quad) =$ an angle

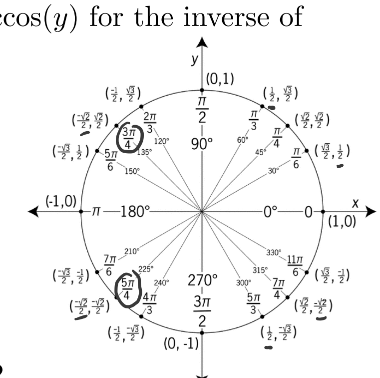
ratio of lengths

We now will write the $\arcsin(y)$ for the inverse of $\sin(x)$ on the domain above and the $\arccos(y)$ for the inverse of $\cos(x)$ on the domain above.

From this we have the following:

(i) Domain: $\arcsin(y)$: $[-1,1]$ $\arccos(y)$: $[-1,1]$

(ii) Range: $\arcsin(y)$: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ $\arccos(y)$: $[0,\pi]$



Learning Outcome # 2: [Using the unit circle to calculate $\arcsin(y)$ and $\arccos(y)$]

Problem 2. Find $\arcsin(y)$ and $\arccos(y)$ for the following values:

(a) 0

$\arcsin(0) = 0$

$\arccos(0) = \pi/2$

(b) 1/2

$\arcsin(1/2) = \frac{\pi}{6}$

$\arccos(1/2) = \pi/3$

(c) $-\sqrt{2}/2$

$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$