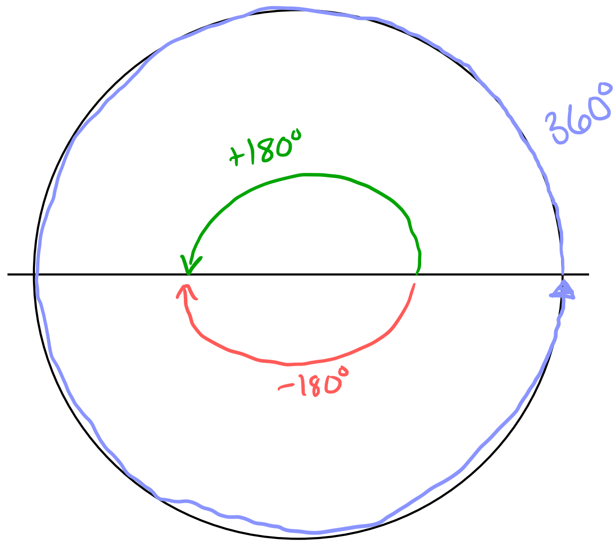
Chapter 5
Section 5.1

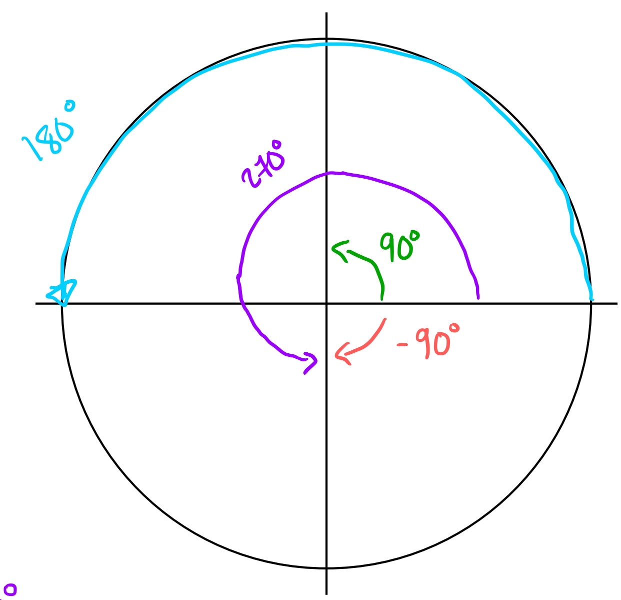
Main Topic # 1: [Degrees v.s. Radians]

Now we are finally in the sections on trigonometry. The first concept we will discuss is measuring angles. The first angle that is the easiest is 360°that is a whole Circle. Next, what do we get when we cut the circle in half?



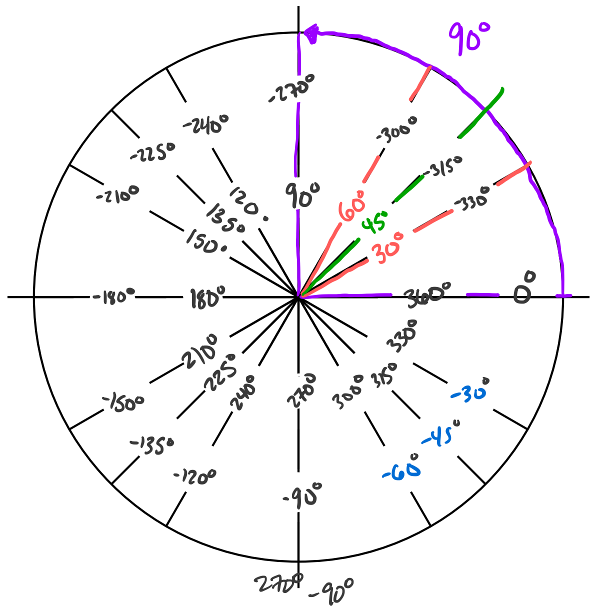
That's right 180°. Also notice when we go counter clockwise we label the degree as positive and when we go clockwise we label the degree as negative.

Now, what happens when we cut that in half?



That's right we get 90°and 270° = 180°+ 90°

What happens if we cut this picture in thirds? What about in fourths?

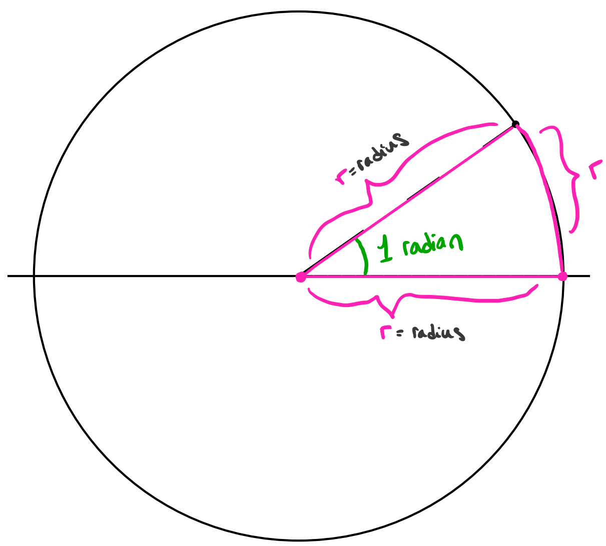


That's right we get 30°, 60°,... etc.

The next concept is **a radian!** Recall that the circumference of a circle with radius r is given by the formula:

$$\text{Circumference} = 2\pi r$$

1 Radian is the size of angle so that the corresponding length of the arc formed by the opening of this angle is r (the radius).



Summary:

The arc length associated to 1 radian = r (the Radius)

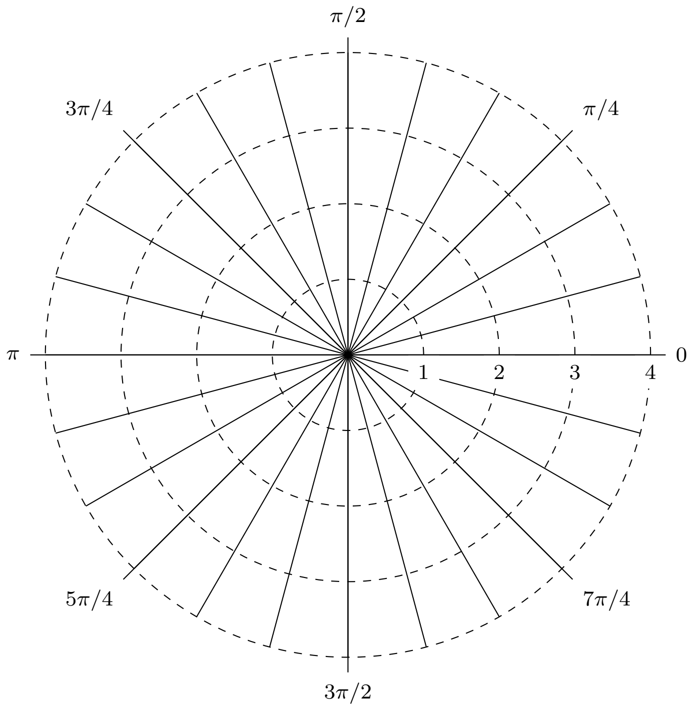
Learning Outcome # 1: [Using the definition of radian to compute arc lengths]

Problem 1. For each of the arcs described below, sketch the arc and determine the length of the arc.

arclength
1 radian $= r$

arclength
2 radians $= 2r$

arclength
of
a radians $= a \cdot r$



(a) $r = 2, \theta = 2\pi$

$2\pi \cdot r = 4\pi$

(b) $r = 4, \theta = \pi/2$

$\frac{2\pi r}{4} = \frac{\pi}{2} \cdot r = \frac{\pi}{2} \cdot 4 = 2\pi$

(c) $r = 1, \theta = \pi/3$

$\frac{2\pi r}{6} = \frac{\pi}{3} \cdot r = \frac{\pi}{3}$

$180^0 = \pi$ radians

(d) $r = 1$, $\theta = \pi \rightarrow$ $\pi \cdot r = \frac{2\pi r}{2}$

Problem 2. Find the length of the arc on the circle of the given radius, defined by the given angle.

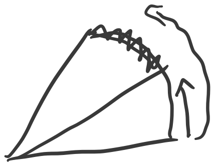
(a) 3 radians on a circle of radius 2.

$3 \cdot r = 6$

(b) 1 radian on a circle of radius 1/4.

$1 \cdot r = 1/4$

Problem 3. Find the length of the arc between the angles $7\pi/6$ and $3\pi/2$ radians, on a circle of radius 2.



$$\frac{3\pi}{2}\cdot 2 - \frac{7\pi}{6}\cdot 2 = 3\pi - \frac{7\pi}{3} = \frac{9\pi - 7\pi}{3} = \frac{2\pi}{3}$$

Problem 4. Find the angle that defines an arc of length 3 on a circle of radius 11.

$$\theta \cdot r = 3 \qquad \theta \cdot 11 = 3 \Rightarrow \theta = \frac{3}{11}$$

what we want

For every π radians

Main Topic # 2: [Converting Radians to Degrees and Degrees to Radians]

Rail Road Tracks

what we want to cancel out

To convert between radians and degrees we see from above that:

$$\frac{\theta \text{ radians}}{1}\cdot\frac{180^\circ}{\pi \text{ radians}} \longleftarrow 180^\circ = \pi \text{ Radians} \longrightarrow \frac{x^\circ}{1}\cdot\frac{\pi \text{ radians}}{180^\circ}$$

we have 180 degrees

Radians and Degrees Conversion

To convert θ Radians to x° we use the following formula:

$$\frac{180 \cdot \theta}{\pi} = x^\circ$$

To convert x° to θ Radians we use the following formula:

$$\frac{\pi \cdot x}{180} = \theta \text{ Radians}$$

Cheat Sheet:

180
18 10
9 2 5 2
3 3

Learning Outcome # 2: [Convert Radians and Degrees]

Problem 5. Convert from degrees to radians:

1. 30°

$$\theta = \frac{\pi \cdot 30}{180} = \frac{\pi}{6} \text{ radians}$$

2. 135°

$$\theta = \frac{\pi \cdot 135}{180} = \frac{3\pi}{4}$$

3. -330°

$$\theta = \frac{\pi \cdot (-330)}{180} = \frac{-11\pi}{6}$$

More Cheats:

135
5 27
3 9
3 3

330
33 10
3 11 5 2

Recall: $x^\circ = \frac{180 \cdot \theta}{\pi}$

Problem 6. Convert from radians to degrees:

1. $5\pi/6$ radians

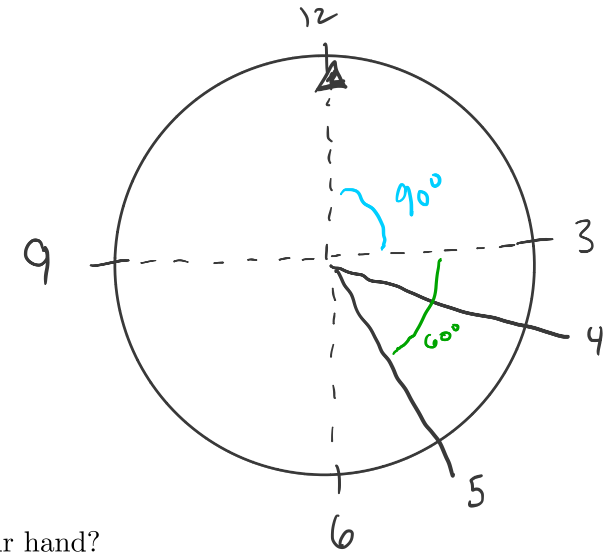
$$x^\circ = \frac{180 \cdot (5\pi/6)}{\pi} = \frac{180}{\cancel{\pi}} \cdot \frac{5\cancel{\pi}}{6} = 150^\circ$$

2. $\pi/4$ radians

$$x^\circ = \frac{180 \cdot (\pi/4)}{\pi} = \frac{180}{\cancel{\pi}} \cdot \frac{\cancel{\pi}}{4} = 45^\circ$$

3. $-4\pi/3$ radians

$$x = \frac{180 \cdot \left(-\frac{4\pi}{3}\right)}{\pi} = \frac{180}{\cancel{\pi}} \cdot \frac{-4\cancel{\pi}}{3} = -240^\circ$$



Problem 7. Suppose an analogue clock strikes 5:00.

1. How many degrees separate the minute hand from the hour hand?

150°

2. How many radians separate the minute hand from the hour hand?

$$\frac{\pi(150)}{180} = \frac{5\pi}{6} \text{ radians}$$

3. How many degrees will separate the minute hand from the hour hand when the clock strikes 8:30?

great Bonus Question

Learning Outcome # 3: [Putting concepts together!]

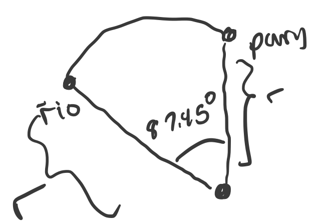
Problem 8. The distance between Paris and Rio de Janeiro is 5,697 miles. If lines were drawn from Paris to Rio to the center of the earth, those lines would meet at an angle of 82.45°. Use this information to approximate the radius of the earth.

$\frac{\pi(82.45)}{180} = 1.438$ radians

Arclength $= r \cdot \theta$ radians

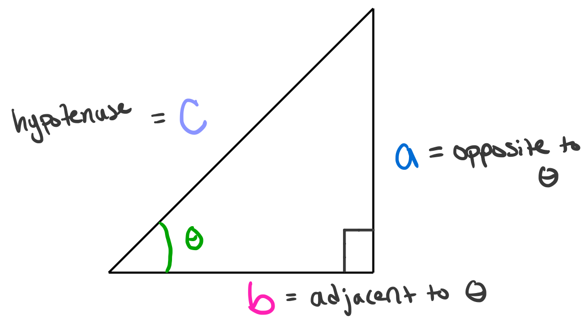
$$\frac{5{,}697}{1.438} = \frac{r \cdot (1.438)}{1.438}$$

$\boxed{r = 3961.75 \text{ miles}}$



Main Topic # 3: [The Basic Points on the Unit Circle]

The unit circle is a circle with radius 1 centered at $(0, 0)$. Therefore the points on the circle have very special meanings corresponding to right triangles. First recall the basic trigometric functions sin and cos.



Sine and Cosine

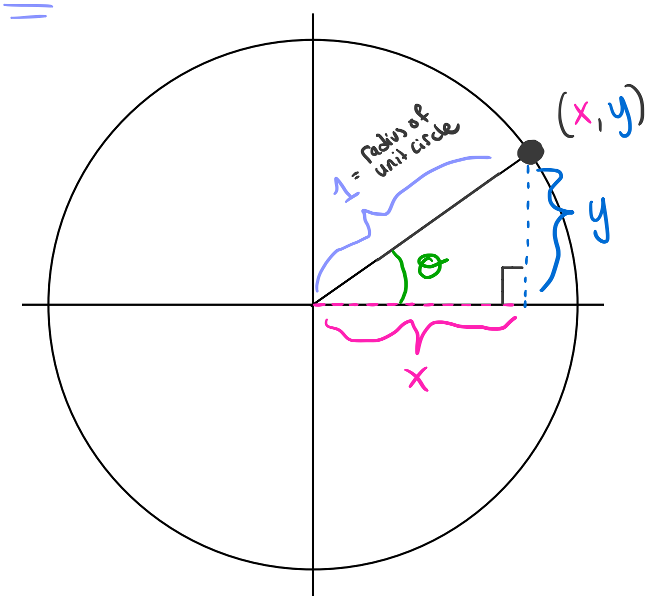
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

But how does this relate to the unit circle?

$x = \cos(\theta)$

$y = \sin(\theta)$



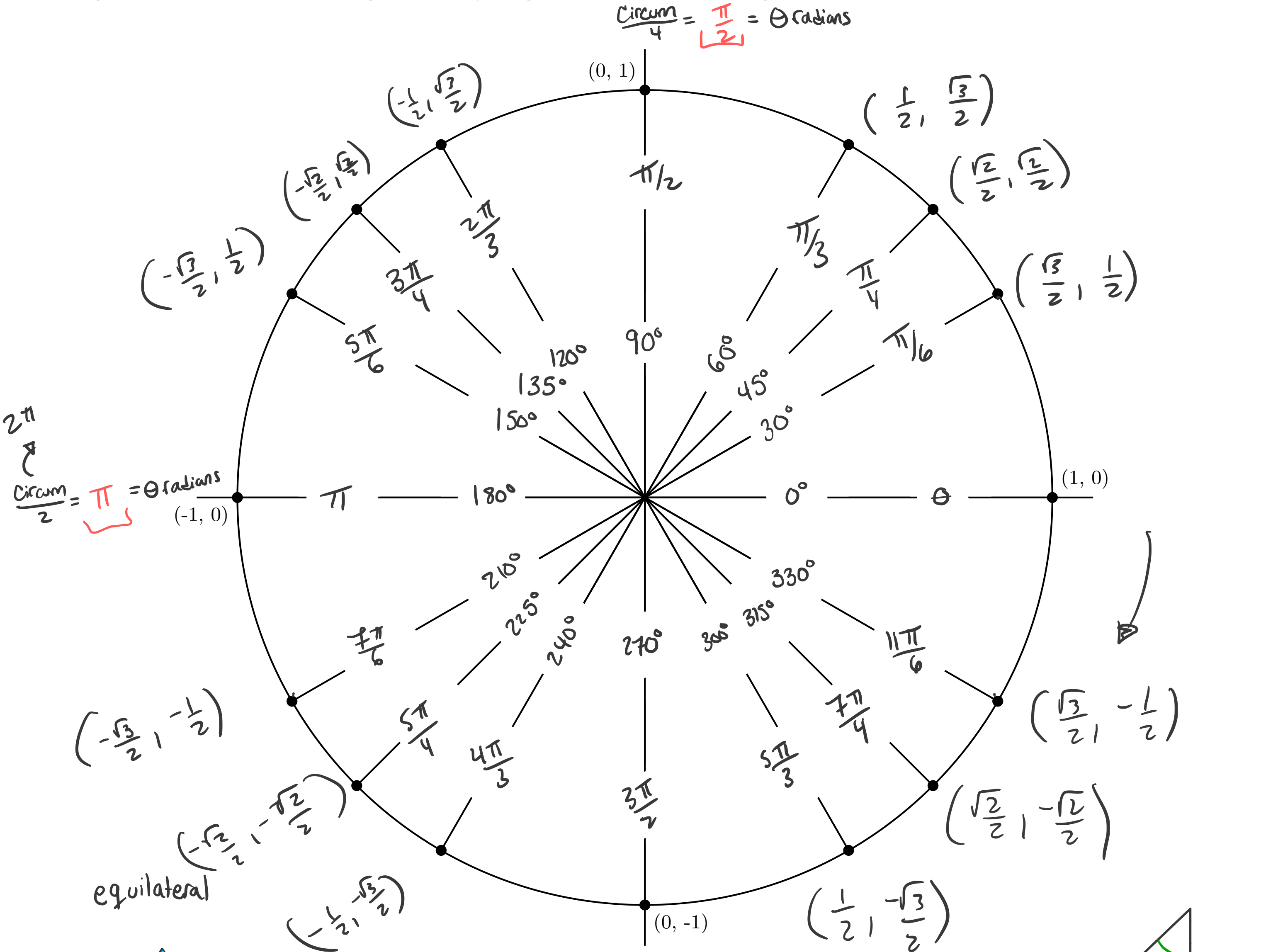
Recall Pythagoras Theorem:

Pythagoras Theorem

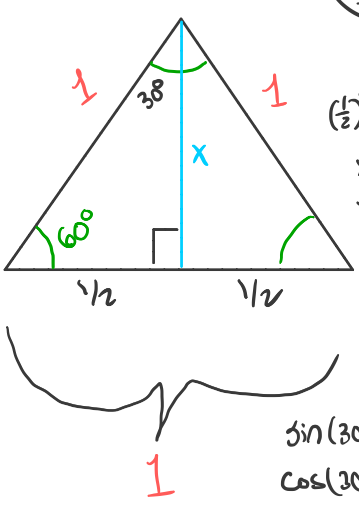
$$a^2 + b^2 = c^2$$

$$(\sin(\theta))^2 + (\cos(\theta))^2 = 1$$

Using basic properties of triangles and Pythagoras theorem we can get the "basic" points on the unit circle:



equilateral



$\left(\frac{1}{2}\right)^2 + x^2 = 1^2$

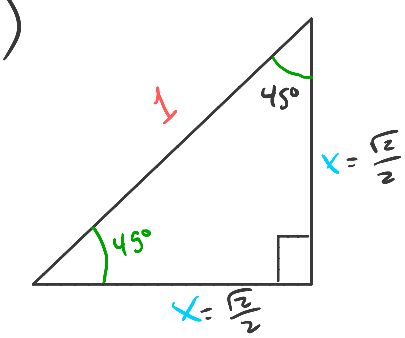
$x^2 = 1 - \frac{1}{4}$

$x^2 = \frac{3}{4}$

$x = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Memorize these!

SOH – CAH – TOA



$\sin(30°) = \frac{1}{2}$ $\quad \sin(60°) = \frac{\sqrt{3}}{2}$

$\cos(30°) = \frac{\sqrt{3}}{2}$ $\quad \cos(60°) = \frac{1}{2}$

$x^2 + x^2 = 1^2$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin(45°) = \frac{\sqrt{2}}{2}$

$\cos(45°) = \frac{\sqrt{2}}{2}$