Math 122 Sections 4.2-4.4 Study Guide

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1 Section 4.2

Second Derivaitve Test:

- **Given:** A function $f(x)$ that is twice differentiable (that is, both its first and second derivatives exist).
- **Goal:** Find the local maxima and minima of $f(x)$.
- **Approach:**
  - Find the critical points of $f(x)$. Recall that the critical points are the x-values where $f'(x) = 0$.
  - If c is a critical point of $f(x)$ and $f''(c) > 0$, then c is a local minimum of $f(x)$.
  - If c is a critical point of $f(x)$ and $f''(c) < 0$, then c is a local maximum of $f(x)$.
  - If c is a critical point of $f(x)$ and $f''(c) = 0$, then it is necessary to use the First Derivative Test to determine if there is a local maximum or local minimum at $x = c$.

Problem 1) Find all local maxima, local minima, and points of inflection for the following functions.

- $f(x) = x^4 - 4x^3$
- $f(x) = -x^3 + 3x^2 + 5$
- $f(x) = x + \frac{4}{x}$.
- $f(x) = x^3$.
- $f(x) = 3x^5 - 5x^3$.

Problem 2) Let $f(x) = x^3 + bx^2 + cx + d$, where b, c, and d are unknown constants. Suppose that there is a critical point at $x = 2$, and an inflection point at $(1, 4)$. Determine b, c, and d.

2 Section 4.3

Problem 3) Find the global maximum and minimum values of the following functions. Unless otherwise specified, assume the domain is $\mathbb{R}$.

- $f(x) = -x^2 + 4x - 5$.
- $f(x) = x + 1/x$ for $x > 0$.
- $f(x) = xe^{-x}$.
- $f(x) = e^{3x} - e^{2x}$.
- $f(x) = x - \ln(x)$ for $x > 0$.
- $f(x) = x^3 - 3x^2$ on $[-1, 3]$.
- $f(x) = x^3 - 3x^2 - 9x + 15$ on $[-5, 4]$.

Problem 4) An individual seeks to enclose a rectangular 1000 square foot plot of land. The fence for the first three sides costs \$22 per foot, and the fence for the remaining side costs \$13 per foot. Determine the minimum cost to enclose the fence.

Problem 5) Determine the minimum value that $x + y$ takes on, given that $xy = 324$ and $x, y > 0$.

Problem 6) Determine the minimum value of $x + 2y$, given that $x^2y = 10$ and $x, y > 0$.

3 Section 4.4

Problem 7) At a price of \$10 per ticket, a musical theater group can fill every seat in the theater, which has a capacity of 1300. For every additional dollar charged, the number of people buying tickets decreases by 50. What ticket price maximizes revenue?

Problem 8) A firm sells a good at \$10 per unit. The cost of producing the good is given by $c(q) = 2q^2 + 5$. What is the quantity that the firm should produce to maximize profit?

Problem 9) A farmer uses x lb of fertilizer per acre, at a cost of \$2/lb. The farmer has a revenue of $R = 700 - 400e^{-x/100}$ dollars per acre. Determine the amount of fertilizer that should be applied per acre to maximize profit.

Problem 10) A landscape architect plans to enclose a 3000 square foot rectangular region. She will use shrubs costing \$45 per foot along three sides, and fencing costing \$20 per foot along the fourth side. What is the maximum revenue the architect can make?