# Math 122 Section 4.1 Study Guide

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**Applying the First Derivative Test:**

- **Given:** A differentiable function $f(x)$.
- **Goal:** Find the local maxima and local minima of $f(x)$.
- **Approach:**
  - Find the critical points of $f(x)$. Recall that the critical points are the $x$-values where $f'(x) = 0$.
  - If the derivative $f'(x)$ changes from positive to negative at a critical point $c$, then $c$ is a local maximum.
  - If the derivative $f'(x)$ changes from negative to positive at a critical point $c$, then $c$ is a local minimum.

**Problem 1)** Find all local maxima and minima for the following functions. Clearly justify your answer using the first derivative test. **It is not enough to find the critical points and stop. You must verify whether a critical point is a maximizer or minimizer.**

- $f(x) = 3x^2 + 2x + 5$.
- $f(x) = -3x^2 + 2x + 5$.
- $f(x) = x^3$.
- $f(x) = 10xe^{3-x^2}$.
- $f(x) = 9x^2 -^3$
- $f(x) = x^4 - 4x^3$
- $f(x) = x + x^{-1}$.

**Problem 2)** Find the constants $a, b$ such that the parabola given by $f(x) = x^2 + ax + b$ has a minimum at the given point. [**Hint:** If there is a local minimum at the point $(x_0, y_0)$, then what must be true of $f'(x_0)$?]

1. $(3, 5)$
2. $(-2, -3)$

**Problem 3)** Find **all** the maxima and minima of the differentiable function $f(x)$, given its derivative:

$$f'(x) = x^2(x + 5)(x + 4)(x - 3)$$

**Clearly justify, in complete sentences, why the relevant points are maxima or minima.** You may use either the first or second derivative test. For your convenience, I have included the table for the first derivative test below:

| | $(x+5)$ | $(x+4)$ | $x^2$ | $(x-3)$ | $f'(x)$ |
|---|---|---|---|---|---|
| $x < -5$ | | | | | |
| $-5 < x < -4$ | | | | | |
| $-4 < x < 0$ | | | | | |
| $0 < x < 3$ | | | | | |
| $x > 3$ | | | | | |