Math 122 Sections 6.4 Example

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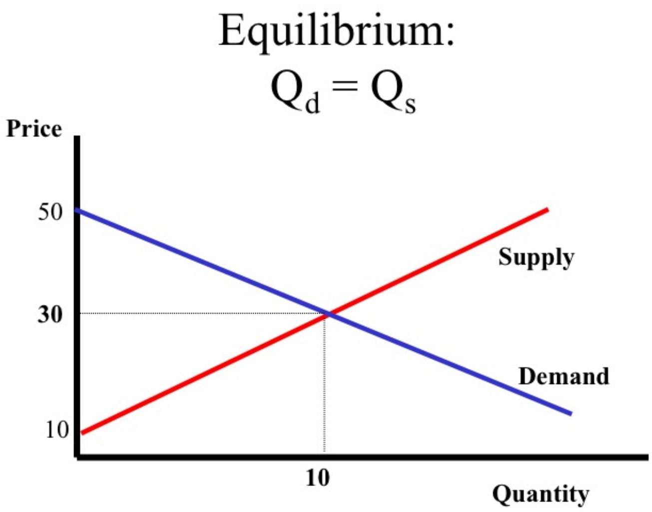
Recall the market equilibrium model. We have:

- A producer supply function $S(p)$, where $S(p)$ is the amount of the good supplied at price p.
- A consumer demand function $Q(p)$, $Q(p)$ is the amount of good demanded at price p.

The *equilibrium price* is the value p^* such that $Q(p) = S(p)$, and the equilibrium quantity q^* is the value of $Q(p^*)$ (or equivocally, $S(p^*)$, as $Q(p^*) = S(p^*)$).

When graphing the supply and demand functions, the x-axis is labeled with quantity and the y-axis is labeled with price. Below is an example of such a graph.



Note that:

- The *consumer surplus* is the area under the demand curve and above the line $y = p^*$.
- The *producer surplus* is the area above the supply curve and below the line $y = p^*$.
- Suppose we have the *inverse demand function* $p_D(q)$ (that is, given $Q(p)$, solve for p). Then the consumer surplus is given by:

$$\int_0^{q^*} p_D(q)\, dq - p^*q^*$$

- Suppose we have the *inverse supply function* $p_S(q)$ (that is, given $S(p)$, solve for p). Then the producer surplus is given by:

$$p^*q^* - \int_0^{q^*} p_S(q)\, dq.$$

Example: Suppose the consumer demand curve is given by $p = 170 - 6q^2$ and equilibrium quantity $q^* = 5$. So consumer surplus is:

$$\int_0^5 (170 - 6q^2)\, dq - 5 * p(5) = \int_0^5 (170 - 6q^2)\, dq - 5 * 20 = 500.$$

Example: Demand curve $p_D = 52 - q^2$. Supply curve: $p_S = 2 + q^2$. Find consumer and producer surplus. We first solve for equilibrium price and quantity. We set $p_S = p_D$ and solve for q; thus, $q^* = 5$. Plugging q^* into either p_S or p_D, we obtain that $p^* = 27$. Now Consumer Surplus is:

$$\int_0^5 (52 - q^2)\, dq - 5(27) = \frac{250}{3} = 83.33,$$

And Producer Surplus is:

$$5(27) - \int_0^5 (2 + q^2)\, dq = \frac{250}{3} = 83.33.$$

Example : Suppose that $Q(p) = 100 - 2p$ and $S(p) = 3p - 50$. We wish to compute the Producer Surplus and Consumer Surplus.

- **Step 1:** We find the equilibrium price and quantity. To do so (Recall Section 1.4), we set $Q(p) = S(p)$. So:

$$\begin{aligned} 100 - 2p &= 3p - 50 \\ 150 &= 5p \\ p^* &= 30 \end{aligned}$$

  And so $q^* = Q(p^*) = Q(30) = 40$.

- **Step 2:** We solve for the inverse supply and inverse demand functions. These are the functions being integrated. To do so, we solve for p for both $Q(p)$ and $S(p)$.

$$\begin{aligned} Q(p) = 100 - 2p &\implies p = 50 - \frac{q}{2} \\ S(p) = 3p - 50 &\implies p = \frac{50}{3} + \frac{q}{3} \end{aligned}$$

- **Step 3:** We now compute the consumer surplus and producer surplus.

  Consumer Surplus:

$$\int_0^{q^*} \left(50 - \frac{q}{2}\right) dq - p^*q^* = \int_0^{40} \left(50 - \frac{q}{2}\right) dq - 30(40) = 400.$$

  Producer Surplus:

$$p^*q^* - \int_0^{q^*} \left(\frac{50}{3} + \frac{q}{3}\right) dq = 30(40) - \int_0^{40} \left(\frac{50}{3} - \frac{q}{3}\right) dq = \frac{800}{3}.$$