Instructions: This quiz is closed book, closed note, and an individual effort. Electronic devices other than approved calculators are not allowed on your person (e.g., no cell phones or calculators with CAS). Answer each question. **Show all work to receive full credit.** Unless the question specifies, you may provide either an exact answer or round to two decimal places.

1. Consider $\displaystyle\int_1^2 \frac{1}{x^2}\,dx$.

(1) **(3 pts)** Using $n = 4$ rectangles and a **left-hand sum**, approximate $\displaystyle\int_1^2 \frac{1}{x^2}\,dx$.

Answer: The base of each rectangle is $\Delta x = \dfrac{2-1}{4} = 0.25$. So the LHS is:

$$0.25\left(\frac{1}{1^2} + \frac{1}{1.25^2} + \frac{1}{1.5^2} + \frac{1}{1.75^2}\right) = 0.6027$$

(2) **(3 pts)** Using $n = 4$ rectangles and a **right-hand sum**, approximate $\displaystyle\int_1^2 \frac{1}{x^2}\,dx$.

Answer: The RHS is:

$$0.25\left(\frac{1}{1.25^2} + \frac{1}{1.5^2} + \frac{1}{1.75^2} + \frac{1}{2^2}\right) = 0.4152$$

(3) **(4 pts)** Compute $\displaystyle\int_1^2 \frac{1}{x^2}\,dx$ exactly (i.e., using our integration rules). Is answer in part (a) an over-estimate or under-estimate? What about your answer in part (b)? Explain.

Answer: We have that:

$$\int_1^2 \frac{1}{x^2}\,dx = -x^{-1}\Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}.$$

As the LHS $= 0.6027$ is larger than $\frac{1}{2}$, the LHS is an overestimate. As the RHS $= 0.4152$ is smaller than $\frac{1}{2}$, the RHS is an underestimate.