Math 122 Supplemental Notes- Section 1.7

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0.1 1.7 Exponential Growth and Decay

Non-Continuous (Discrete) Compound Interest Model: We have the following setup:

- P: Principal Amount
- r: Rate
- n: Number of times compounded per year
- t: total number of years

Then:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Example: If \$2500 is invested at 6% interested compounded daily, then how long would it take for the investment to double?

Answer: Here, we want to solve for time t. As interest is compounded daily, we have:

$$P(t) = 2500\left(1 + \frac{.06}{365}\right)^{365t}.$$

Since we are interested in doubling time, we care when $P(t) = 5000$. So we have that:

$$5000 = 2500\left(1 + \frac{0.06}{365}\right)^{365t}$$
$$2 = \left(1 + \frac{0.06}{365}\right)^{365t}$$
$$\ln(2) = 365t \cdot \ln\left(1 + \frac{0.06}{365}\right)$$
$$t = \frac{\ln(2)}{365 \cdot (1 + \frac{0.06}{365})}$$

So t is roughly 11.553 years. So the investment will double in 11 years and 202 days.

Continuous Compound Interest: Our model for continuous compound interest is similar to the discrete compound interest model. Here, we have P, the initial amount; and r, the interest rate. Again, t is the total number of years. Our equation is the familiar $A(t) = Pe^{rt}$.

Example: If \$2500 is invested at 6% interested compounded continuously, then how long would it take for the investment to double?

Answer: The logic is the same as in the previous example, though with a different equation. Here, we have:

$$5000 = 2500e^{.06t}$$
$$2 = e^{.06t}$$
$$\ln(2) = 0.06t$$
$$t = \ln(2)/.06 = 11.552 \text{ years.}$$

Radioactive Decay

Example: Suppose we have a spill of radioactive iodine. The initial radiation was about 2.4 millirems/hour (four times acceptable limit of 0.6 millirems/hour). So the EPA ordered evacuation of area. The radiation decays at continuous hourly rate of $r = -0.004$.

(a) What is radiation level after 24 hours?

Answer: As the decay is continuous, our formula is given by $P(t) = 2.4e^{-.004t}$. So the amount of radiation after 24 hours is:

$$P(24) = 2.4e^{-.004*24} = 2.18 \text{ millirems/hour.}$$

(b) How many hours until acceptable limit?

Answer: Recall that the acceptable limit is 1/4 the inital amount. So we have that:

$$\begin{aligned} 0.6 &= 2.4e^{-.004t} \\ 1/4 &= e^{-.004t} \\ \ln(1/4) &= -0.004t \\ t &= \ln(1/4)/-0.004 = 346.57 \text{ hours.} \end{aligned}$$

(c) What is half-life of radiation?

Answer: For half-life and doubling problems, we are focused on *relative change*. So we can take the initial amount to be 1 (as we start with 100% of the item) and the final amount to be the desired end percentage. For half-life, the final amount would be 1/2. So for our radioactive iodine model, we have:

$$\begin{aligned} 1/2 &= e^{-0.004t} \\ \ln(1/2) &= -0.004t \\ t &= \ln(1/2)/(-0.004) = 173.29 \text{ hours.} \end{aligned}$$