Math 122 Sections 2.1-2.3 Study Guide

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1 Section 2.1

1) For each of the following, graph a differentiable function $f(x)$ satisfying (at a point c):

(a) $f(c) > 0$ and $f'(c) > 0$

(b) $f(c) > 0$ and $f'(c) = 0$

(c) $f(c) > 0$ and $f'(c) < 0$

(d) $f(c) = 0$ and $f'(c) > 0$

(e) $f(c) = 0$ and $f'(c) = 0$

(f) $f(c) = 0$ and $f'(c) < 0$

(g) $f(c) < 0$ and $f'(c) > 0$

(h) $f(c) < 0$ and $f'(c) = 0$

(i) $f(c) < 0$ and $f'(c) < 0$

2) Suppose $f(x) = x^2$. Which of the following properties from 1(a)-1(d) does $f(x)$ satisfy at:

- $x = 0$
- $x = -2$
- $x = 5$

3) Repeat problem (2) for $f(x) = x^5$.

4) Repeat problem (2) for $f(x) = \sqrt{x}$, for the values $x = 0$ and $x = 5$ only.
5) Repeat problem (2) for $f(x) = \ln(x)$, for the values $x = 1$, $x = 1/e$, and $x = 5$.

2 Sections 2.2 and 2.3

Recall the example from class:

t (min)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$c(t)$ (mg/cc)	0.84	0.89	0.94	0.98	1.00	1.00	0.97	0.90	0.79	0.63	0.41

As we do not have an equation for a function, we cannot compute the derivative using our derivative rules from class. Therefore, the best we can hope to do is estimate the rate of change at a given point. To this end, we have three estimates of interest: the left-hand estimate, the right-hand estimate, and the two-hand (or two-sided) estimate. Each of these estimates is an **average rate of change**. That is, we are using the **slope formula**. The difference amongst these estimates comes down to the endpoints in question.

(a) **Left-Hand Estimate:** Suppose we want to estimate $c'(t)$. Here, we use the endpoint $c(t)$, as well as the endpoint immediately before $c(t)$ (which we denote $c(t-h)$). Here, h represents the distance between t and the point immediately preceeding t. The left-hand estimate is just the average rate of change between $c(t)$ and $c(t-h)$:

$$c'(t) \approx \frac{c(t) - c(t-h)}{t - (t-h)} = \frac{c(t) - c(t-h)}{h}.$$

Example: Suppose we want to estimate $c'(0.2)$ using the left-hand estimate. Here, $t = 0.2$, so $t-h = 0.1$ (thus, $h = 0.1$). The left-hand estimate of $c'(0.2)$ is:

$$\frac{c(0.2) - c(0.1)}{0.2 - 0.1} = \frac{0.94 - 0.89}{0.1} = 0.5.$$

(b) **Right-Hand Estimate:** Suppose we want to estimate $c'(t)$. Here, we use the endpoint $c(t)$, as well as the endpoint immediately before $c(t)$ (which we denote $c(t+h)$). Here, h represents the distance between t and the point immediately preceeding t. The right-hand estimate is just the average rate of change between $c(t+h)$ and $c(t)$:

$$c'(t) \approx \frac{c(t+h) - c(t)}{(t+h) - t} = \frac{c(t+h) - c(t)}{h}.$$

Example: Suppose we want to estimate $c'(0.2)$ using the right-hand estimate. Here, $t = 0.2$, so $t+h = 0.3$ (thus, $h = 0.1$). The right-hand estimate of $c'(0.2)$ is:

$$\frac{c(0.3) - c(0.2)}{0.3 - 0.2} = \frac{0.98 - 0.94}{0.1} = 0.4.$$

(c) **Two-Sided Estimate:** Suppose we want to estimate $c'(t)$. Here, we use the endpoints immediately before $c(t)$ (which we denote $c(t-h)$) and immediately after $c(t)$ (which we denote $c(t+h)$). The two-sided estimate of $c'(t)$ is the average rate of change between $c(t+h)$ and $c(t-h)$:

$$c'(t) \approx \frac{c(t+h) - c(t-h)}{(t+h) - (t-h)} = \frac{c(t+h) - c(t-h)}{2h}.$$

Example: Suppose we want to estimate $c'(0.2)$ using the two-sided estimate. Here, we use $t = 0.2$, so $t - h = 0.1$ and $t + h = 0.3$ (thus, $h = 0.1$). The two-sided estimate of $c'(0.2)$ is:

$$\frac{c(0.3) - c(0.1)}{0.3 - 0.1} = \frac{0.98 - 0.89}{0.3 - 0.1} = \frac{0.09}{0.2} = 0.45.$$

6) Consider the following table:

t	2.7	3.2	3.7	4.2	4.7	5.2	5.7	6.2
$c(t)$	3.4	4.4	5	5.4	6	7.4	9	11

(a) Provide a left-hand estimate of $c'(4.2)$. [**Note:** You can repeat this problem for any value of t except $t = 2.7$.]

(b) Provide a right-hand estimate of $c'(4.2)$. [**Note:** You can repeat this problem for any value of t except $t = 6.2$.]

(c) Provide a two-sided estimate of $c'(4.2)$. [**Note:** You can repeat this problem for any value of t except $t = 2.7$ and $t = 6.2$.]

(d) Using your estimate of $c'(4.2)$ from part (c), determine the equation of the line tangent to $c(t)$ at $t = 4.2$.

(e) Using your tangent line from part (d), estimate $c(4.4)$ (that is, plug 4.4 as the input for your tangent line).

7) For each problem, find the tangent line and use it to approximate the function at the given x-value.

(a) $f(4) = 5$ and $f'(4) = 7$. Approximate $f(4.02)$. Then approximate $f(3.92)$.

(b) $f(5) = 3$ and $f'(5) = -2$. Approximate $f(5.03)$.

(c) $f(2) = -4$ and $f'(2) - 3$. Approximate $f(1.95)$.

(d) Find the line tangent to $f(x) = \exp(x^2)$ at $x = 0.3$, and use it to approximate $f(0.5)$.

(e) Find the line tangent to $f(x) = \ln(3x)$ at $x = 2$, and use it to approximate $f(2.4)$.