MATH 550 Spring, 2008 Quiz #7 Name: M^2

1. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for $\mathbf{F} = (9xy^2, x^2y, z)$ for S the closed cylinder, oriented by **outward normals** and bounded by $x^2+y^2=4$, $z=-2$, and $z=3$.

$$\iint_S \vec{F}\cdot d\vec{S} \underset{\text{Gauss}}{=} \iiint_D \vec{\nabla}\cdot\vec{F}\, dV \qquad D = \text{interior of } S$$

$$= \iiint_D (9y^2+x^2+1)\,dV = \int_0^{2\pi}\int_0^{2}\int_{-2}^{3} (9r^2\sin^2\theta + r^2\cos^2\theta + 1)\, dz\, r\, dr\, d\theta$$

$$= 5\int_0^{2\pi}\int_0^{2} (8r^3\sin^2\theta + r^3 + r)\, dr\, d\theta$$

(Note: $x^2+y^2=r^2 \neq 4$ inside D)

$$= 5\int_0^{2\pi} (32\sin^2\theta + 4 + 2)\, d\theta$$

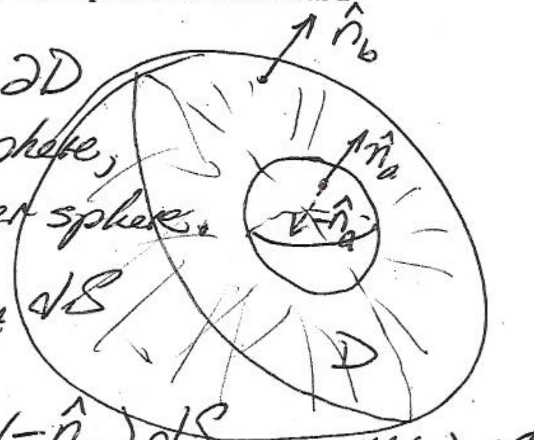
$$= 5(6)(2\pi) + (5)(32)\left(\tfrac{1}{2}\theta - \frac{\cos 2\theta}{4}\right)\Big|_0^{2\pi}$$

$$= (5)(12)(\pi) + 5(32)(\pi)$$

$$= 220\pi$$ (no guarantee of arithmetic accuracy)

2. (5 points) A small sphere S_a of radius a is inside a larger sphere S_b of radius b; the centers may not be at the same point however. Suppose $\vec{\nabla}\cdot\mathbf{F} = 0$ throughout the region D between the two spheres. How are the integrals $\int_{S_a} \mathbf{F}\cdot\hat{\mathbf{n}}_a\, dS$ and $\int_{S_b} \mathbf{F}\cdot\hat{\mathbf{n}}_b\, dS$ related, if $\hat{\mathbf{n}}_a$ and $\hat{\mathbf{n}}_b$ represent the **outward normals** to each sphere? Briefly explain.

Note that $\hat{n}_{out}$ for $S = \partial D$ is $\hat{n}_b$ on the outer sphere, but $-\hat{n}_a$ on the inner sphere.



$$0 = \iiint_D \underbrace{\vec{\nabla}\cdot\vec{F}}_{=0}\, dV = \iint_{\partial D} \vec{F}\cdot\hat{n}_{out}\, dS$$

$$= \iint_{S_b} \vec{F}\cdot\hat{n}_b\, dS + \iint_{S_a} \vec{F}\cdot(-\hat{n}_a)\, dS$$

pull $(-)$ out of $\iint$

Hence $$\iint_{S_a} \vec{F}\cdot\hat{n}_a\, dS = \iint_{S_b} \vec{F}\cdot\hat{n}_b\, dS$$