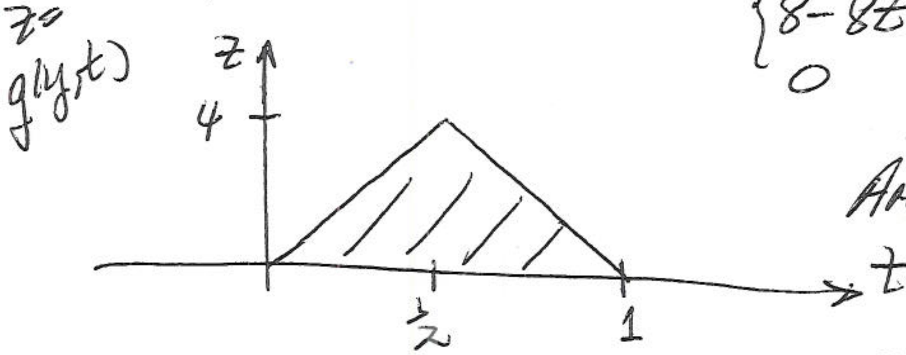
Bonus problem: here are some snapshots of $g(y,t)$:

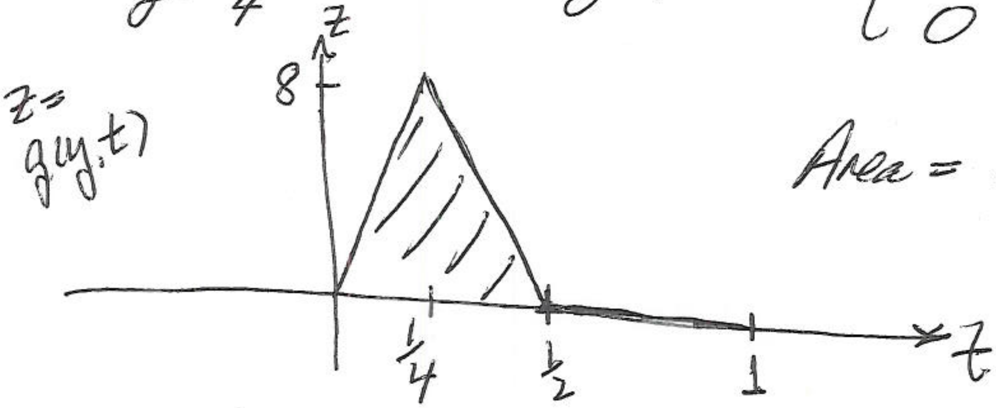
$y = \frac{1}{2}$ $\qquad g(\frac{1}{2}, t) = \begin{cases} 8t & 0 \le t \le \frac{1}{2} \\ 8 - 8t & \frac{1}{2} \le t \le 1 \\ 0 & t = 1 \end{cases}$



Area $= \frac{1}{2} BH = 2$

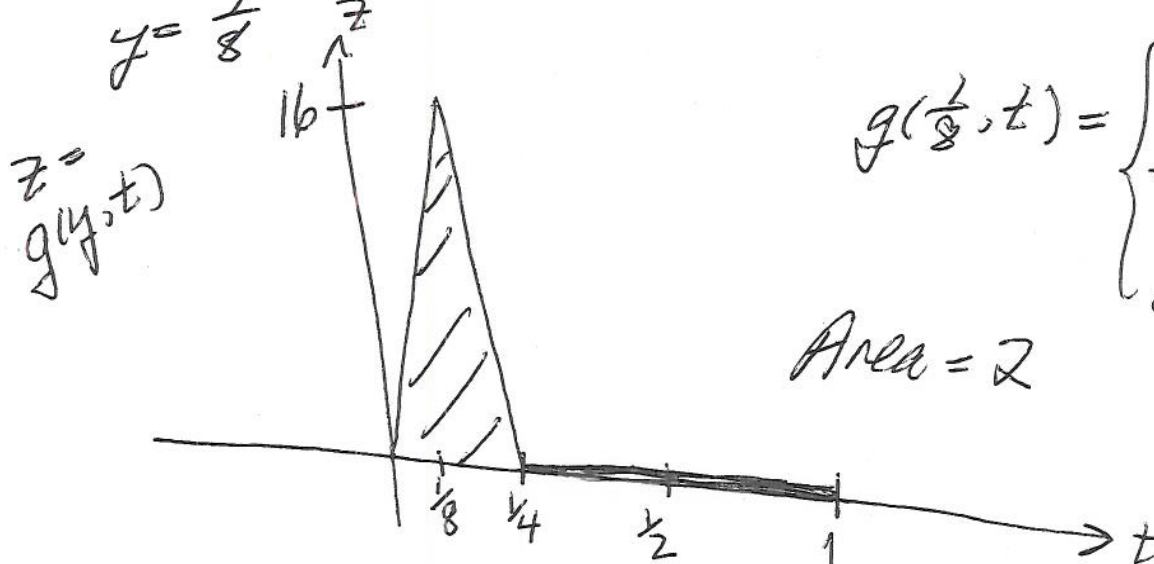
$g(\frac{1}{4}, t) = \begin{cases} 32t & 0 \le t \le \frac{1}{4} \\ 16 - 32t & \frac{1}{4} \le t \le \frac{1}{2} \\ 0 & \frac{1}{2} \le t \le 1 \end{cases}$

$y = \frac{1}{4}$



Area = 2

$y = \frac{1}{8}$



$g(\frac{1}{8}, t) = \begin{cases} 128t & 0 \le t \le \frac{1}{8} \\ 32 - 128t & \frac{1}{8} \le t \le \frac{1}{4} \\ 0 & \frac{1}{4} \le t \le 1 \end{cases}$

Area = 2

Peaks move left, get narrower & narrower, higher & higher (so area stays constant at 2!)

Notice that for *fixed* t, as $y \to 0$, the positive part of the graph will eventually slide to the left of t, and from then on $g(y,t) = 0$ for smaller and smaller y values. Hence $\lim_{y\to 0} g(y,t) = 0$, and so

$$\int_0^1 \left(\lim_{y\to 0} g(y,t)\right) dt = \int_0^1 0\,dt = 0.$$

On the other hand for *fixed* y,

$$\int_0^1 g(y,t)\,dt = \text{Area under peak} = 2,$$

so $\displaystyle\lim_{y\to 0} \int_0^1 g(y,t)\,dt = \lim_{y\to 0} 2 = 2.$

Intuition: increasing height of peak compensates for narrowing base. Evidently we would need $g(y,t)$ to be defined for $y=0$, and for this to happen we should have the values of $g(y,t)$ bounded for $y>0$. Then the areas would go to 0 as the function values go to zero.