MATH 544   Spring, 2009   Exam #1   Name:____________________

Note! For full credit you must show sufficient work to support your answer. Use the space you need for parts a., b., etc., but label clearly which part of a problem you are doing where. In Part I (yellow paper) you may not use a calculator, and I expect to see a clear reckoning of the row reduction. In part II you may use a calculator for any arithmetic operations. There are 100 points. *Good luck!*

Part I.

1. (30 points) Let $A = \begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 2 & -8 & 2 \\ 1 & 0 & -1 & 1 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

   a. What condition(s) must be imposed on the entries of $\mathbf{b}$ so that $\mathbf{b}$ is in the span of the columns of A? Give a vector in $\mathbb{R}^3$ that is not in the span of the columns of A, or explain why this span is all of $\mathbb{R}^3$.

   b. Find all solutions of $A\mathbf{x} = \mathbf{0}$.

Part II. **Name:**____________________

2. (20 points) We are given that $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $T(\mathbf{v}_1) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, and $T(\mathbf{v}_2) = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.
   a. Compute $T(\mathbf{v}_1 + \mathbf{v}_2)$ and $T(-3\mathbf{v}_1 + \mathbf{v}_2)$.
   b. Compute the matrix A for which $T = L_A$ (Hint: use part (a) together with computations of $\mathbf{v}_1 + \mathbf{v}_2$ and $-3\mathbf{v}_1 + \mathbf{v}_2$).

3. (10 points) Let $T\colon \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ be vectors in $\mathbb{R}^n$, and assume that $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is an independent set in $\mathbb{R}^m$. Show that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an independent set in $\mathbb{R}^n$.

4. (30 points) Let $A = \begin{bmatrix} -2 & 4 & 10 & 1 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & -2 & 0 \\ -3 & 3 & 9 & 1 \end{bmatrix}$.

a. Compute the reduced row echelon form of A.

b. Is the set of columns of A independent? If so, explain; if not express one column as a linear combination of the others, and determine the largest set of columns that is independent.

c. $A\mathbf{x} = \begin{bmatrix} 4 \\ -1 \\ 0 \\ 5 \end{bmatrix}$ has a solution $\mathbf{v}$, which you need not compute. Are there any other solutions? If not, explain; if so, describe them explicitly.

5. (10 points) Let $A = \begin{bmatrix} -3 & 1 \\ 8 & 4 \end{bmatrix}$.

a. Find all solutions of the system $A\mathbf{x} = -4\mathbf{x}$ for $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$.

b. (Bonus) There is exactly one other value of c for which $A\mathbf{x} = c\mathbf{x}$ is consistent for some $\mathbf{x} \neq \mathbf{0}$. Determine this value of c.