3.6 #1 $\quad y'' - 5y' + 6y = 2e^{t}$

Find homog solution y_H for $y'' - 5y' + 6 = 0$.

$$r^2 - 5r + 6 = 0$$
$$(r-3)(r-2) = 0$$
$$r = 3 \quad r = 2$$
$$y_1 = e^{3t} \quad y_2 = e^{2t}$$

$$W = \begin{vmatrix} e^{3t} & e^{2t} \\ 3e^{3t} & 2e^{2t} \end{vmatrix} = -e^{5t}$$

Seek u_1, u_2 so that $y_p = u_1 y_1 + u_2 y_2$

$$\begin{cases} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = 2e^{t} \end{cases}$$

$$-2 \;\Big|\; -3 \begin{cases} u_1' e^{3t} + u_2' e^{2t} = 0 \\ u_1' 3e^{3t} + u_2' 2e^{2t} = 2e^{t} \end{cases}$$

$$2u_2' e^{2t} - 3u_2' e^{2t} = 2e^{t}$$

$$-u_2' = 2e^{t} e^{-2t} = 2e^{-t}$$

$$u_2 = 2\int -e^{-t}\,dt = 2e^{-t}$$

Similarly,

$$-2u_1' e^{3t} + u_1' 3e^{3t} = 2e^{t}$$

$$+u_1' = 2e^{t} e^{-3t} = 2e^{-2t}$$

$$u_1 = \underline{[illegible]}\, e^{-2t}$$

$y = y_H + y_p = c_1 e^{3t} + c_2 e^{2t} + u_1 e^{3t}$
$+ u_2 e^{2t}$

$= c_1 e^{3t} + c_2 e^{2t} +$ [illegible] $e^{t} + 2e^{t}$

$= c_1 e^{3t} + c_2 e^{2t} +$ [illegible] e^{t}