Math 520 Spring 2012 Quiz #7 & #8 Name: Key

Quiz #7 1. Let $A = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix}$. Find the solution to the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Note that $\lambda = -3$ is the only eigenvalue of A.

Find eigenvectors: $(A+3I)\vec{v} = \vec{0}$ $\begin{pmatrix} 4 & -4 \\ 4 & -4 \end{pmatrix}\vec{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$4v_1 - 4v_2 = 0$

$v_1 = v_2$ so $\vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. & $\vec{x}^{(1)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}$

To find a second lin. indep. soln: $\vec{x}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-3t} + \vec{u}e^{-3t}$

where $(A+3I)\vec{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$: $\begin{pmatrix} 4 & -4 \\ 4 & -4 \end{pmatrix}\vec{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ so $4u_1 - 4u_2 = 1$

$u_1 - u_2 = \frac{1}{4}$

$\therefore \vec{u} = \begin{pmatrix} u_2 + \frac{1}{4} \\ u_2 \end{pmatrix}$

with $u_2 = 0$: $\vec{u} = \begin{pmatrix} 1/4 \\ 0 \end{pmatrix}$.

$\therefore \vec{x}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-3t} + \begin{pmatrix} 1/4 \\ 0 \end{pmatrix} e^{-3t}$

To satisfy the IC: $\vec{x} = c_1\vec{x}^{(1)} + c_2\vec{x}^{(2)} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-3t} + c_2\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}t + \begin{pmatrix} 1/4 \\ 0 \end{pmatrix}\right)e^{-3t}$

$\vec{x}(0) = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2\begin{pmatrix} 1/4 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$c_1 + \frac{1}{4}c_2 = 3 \implies c_2 = 4$

$c_1 = 2$

$\therefore \vec{x}(t) = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{-3t} + 4\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}t + \begin{pmatrix} 1/4 \\ 0 \end{pmatrix}\right)e^{-3t}$

$= \left(\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 4t+1 \\ 4t \end{pmatrix}\right)e^{-3t} = \begin{pmatrix} 4t+3 \\ 4t+2 \end{pmatrix}e^{-3t}$

Quiz #8 2. Find the general solution to $y''' - y'' - y' + y = 0$.

$y = e^{rt}$: $r^3 - r^2 - r + 1 = 0$

$(r-1)(r^2-1) = 0$

$(r-1)(r-1)(r+1) = 0$

$(r-1)^2(r+1) = 0$

$r = 1, 1, -1$

$y_1 = e^t$

$y_2 = te^t$

$y_3 = e^{-t}$

$y = c_1e^t + c_2te^t + c_3e^{-t}$.