MATH 520 **Spring, 2012** **Quiz #4** **Name:** M^2

1. Let $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$. Verify that $\mathbf{x}^{(1)} = \begin{bmatrix} e^{2t} \\ -e^{2t} \end{bmatrix}$ and $\mathbf{x}^{(2)} = \begin{bmatrix} 3e^{4t} \\ -e^{4t} \end{bmatrix}$ are solutions to $\mathbf{x}' = A\mathbf{x}$. Are these solutions independent for $-\infty < t < \infty$?

$$\vec{x}^{(1)\prime} = \begin{bmatrix} 2e^{2t} \\ -2e^{2t} \end{bmatrix} = Ax^{(1)} = \begin{bmatrix} 5e^{2t} - 3e^{2t} \\ -e^{2t} - e^{2t} \end{bmatrix}$$

$$\vec{x}^{(2)\prime} = \begin{bmatrix} 12e^{4t} \\ -4e^{4t} \end{bmatrix} = A\vec{x}^{(2)} = \begin{bmatrix} 15e^{4t} - 3e^{4t} \\ -3e^{4t} - e^{4t} \end{bmatrix}$$

$$W = \begin{vmatrix} e^{2t} & 3e^{4t} \\ -e^{2t} & -e^{4t} \end{vmatrix} = -e^{6t} + 3e^{6t} = 2e^{6t} \neq 0$$

on $(-\infty, \infty)$

2. Find the eigenvalues and eigenvectors for the matrix $\begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$. $= A$

$$\begin{vmatrix} 1-r & \sqrt{3} \\ \sqrt{3} & -1-r \end{vmatrix} = -1 + r^2 - 3 = r^2 - 4$$

$$= (r-2)(r+2)$$

$r = 2, r = -2$

$r=2$ $\left[\begin{array}{cc|c} -1 & \sqrt{3} & 0 \\ \sqrt{3} & -3 & 0 \end{array}\right] \xrightarrow{\sqrt{3}R_1 + R_2} \left[\begin{array}{cc|c} -1 & \sqrt{3} & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\xrightarrow{-R_1} \left[\begin{array}{cc|c} 1 & -\sqrt{3} & 0 \\ 0 & 0 & 0 \end{array}\right]$ $\vec{\xi}^{(1)} = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$ $r=2$

Check: $A\vec{\xi}^{(1)} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} = \begin{bmatrix} 2\sqrt{3} \\ 2 \end{bmatrix} = 2\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$

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1. Let $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$. Verify that $\mathbf{x}^{(1)} = \begin{bmatrix} e^{2t} \\ -e^{2t} \end{bmatrix}$ and $\mathbf{x}^{(2)} = \begin{bmatrix} 3e^{4t} \\ -e^{4t} \end{bmatrix}$ are solutions to $\mathbf{x}' = A\mathbf{x}$. Are these solutions independent for $-\infty < t < \infty$?

2. Find the eigenvalues and eigenvectors for the matrix $\begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$.

$$r = -2 \quad \left[\begin{array}{cc|c} 3 & \sqrt{3} & 0 \\ \sqrt{3} & 1 & 0 \end{array}\right] \xrightarrow{-\sqrt{3}R_2 + R_1} \left[\begin{array}{cc|c} 0 & 0 & 0 \\ \sqrt{3} & 1 & 0 \end{array}\right]$$

$$\vec{\xi}^{(2)} = \begin{bmatrix} -\frac{1}{\sqrt{3}} \\ 1 \end{bmatrix} \qquad \longrightarrow \left[\begin{array}{cc|c} \sqrt{3} & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Check $$A\vec{\xi}^{(2)} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}\begin{bmatrix} -\frac{1}{\sqrt{3}} \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{3}} + \sqrt{3} \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2/\sqrt{3} \\ -2 \end{bmatrix} = -2\begin{bmatrix} -\frac{1}{\sqrt{3}} \\ 1 \end{bmatrix}$$