MATH 520 Spring, 2012 Quiz #2 Name: M^2

1. Solve the differential equation $y' = (1-2x)/y$ with initial condition $y(1) = -2$ explicitly. For which values of x is the solution defined?

$$\frac{dy}{dx} = \frac{1-2x}{y}$$

$$x = \frac{-1 \pm \sqrt{1-4(-1)2}}{2} = (-1 \pm 3)/2 = 1, -2$$

$$\int y\,dy = \int (1-2x)\,dx$$

$$y(1) = -2$$

$$\frac{1}{2}y^2 = x - x^2 + C \leftarrow 2 = C$$

$$y^2 = 2x - 2x^2 + 4$$

$$y = -\sqrt{2x - 2x^2 + 4}$$ Minus because $y(1) = -2$

Solution is defined for $2 + x - x^2 \geq 0$

$x = 0$ at $x = 1$, $x = -2$.

Domain of solution $-2 \leq x \leq 1$.

2. Find the general solution to the equation $y'' - y' - 6y = 0$. Then determine the unknown constants by using the IC $y(0) = 3$ and $y'(0) = 4$.

Char eqn $r^2 - r - 6 = 0$

$(r-3)(r+2) = 0$ $r = 3$, $r = -2$

$y_1 = e^{3t}$, $y_2 = e^{-2t}$

General solution $y = C_1 e^{3t} + C_2 e^{-2t}$

Then $y' = C_1 3e^{3t} + C_2(-2)e^{-2t}$

At $t=0$, $\begin{cases} 3 = C_1 + C_2 \\ 4 = 3C_1 - 2C_2 \end{cases}$ or $\begin{cases} 6 = 2C_1 + 2C_2 \\ 4 = 3C_1 - 2C_2 \end{cases}$

$10 = 5C_1$, $C_1 = 2$, so $C_2 = 1$.