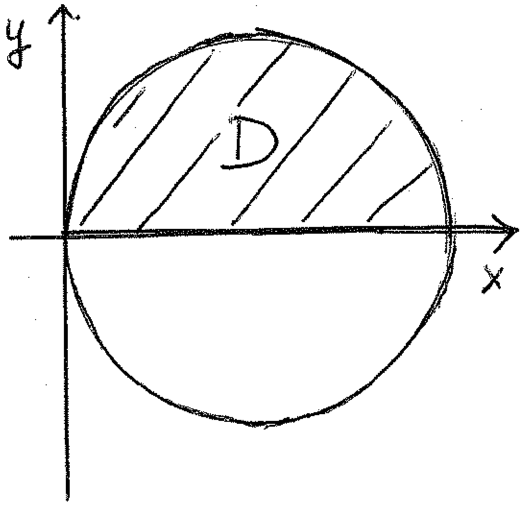
MATH 241 Spring, 2010 Quiz #7 Name: M^2

For full credit you must show sufficient work that the method of obtaining your answer is clear.

1. Fill in the limits of integration and the corresponding dA in polar coordinates for the double integral $\iint_D \sin\theta \, dA$, where D is the region in the first quadrant that lies inside the circle $r = 3\cos\theta$. Do NOT compute.

$$\int_0^{\pi/2} \int_0^{3\cos\theta} \sin\theta \, r \, dr \, d\theta$$



OR

$$V = \int_0^6 \int_0^{12-2y} \int_0^{4-\frac{1}{3}x-\frac{2}{3}y} dz \, dx \, dy$$

$$= \int_0^6 \int_0^{12-2y} \left(4 - \tfrac{1}{3}x - \tfrac{2}{3}y\right) dx \, dy$$

2. SET UP, but DO NOT compute, either a double or a triple integral that gives the volume in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) bounded by the plane $x + 2y + 3z = 12$ and the three coordinate planes.

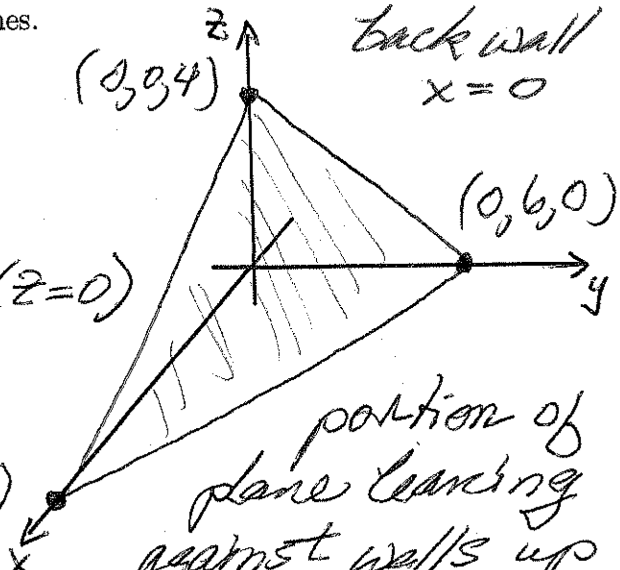
side wall $y=0$, back wall $x=0$

$$3z = 12 - x - 2y$$

$$z = 4 - \tfrac{1}{3}x - \tfrac{2}{3}y$$

Intersection with floor ($z=0$) is $x + 2y = 12$ or

$$y = 6 - \tfrac{1}{2}x$$



portion of plane leaning against walls up from floor ($z=0$)

$$V = \int_0^{12} \int_0^{6-\frac{1}{2}x} \int_0^{4-\frac{1}{3}x-\frac{2}{3}y} dz \, dy \, dx$$

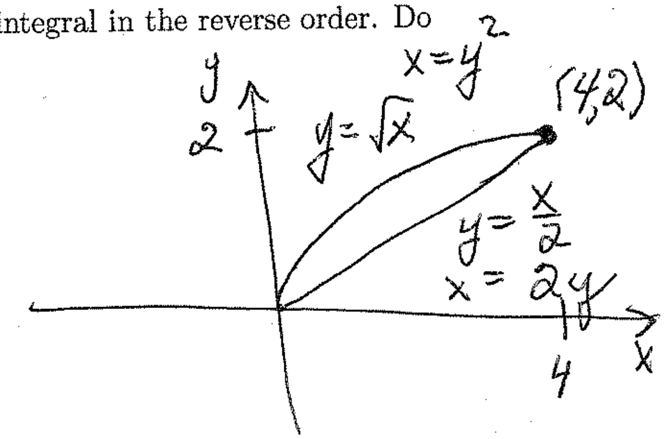
(over →)

$$= \int_0^{12} \int_0^{6-\frac{1}{2}x} \left(4 - \tfrac{1}{3}x - \tfrac{2}{3}y\right) dy \, dx$$

3. Rewrite $\int_0^4 \int_{x/2}^{\sqrt{x}} f(x,y)\,dy\,dx$ as an iterated integral in the reverse order. Do NOT compute.

$$\int_0^2 \int_{y^2}^{2y} f(x,y)\,dx\,dy$$

left (lower "x") to right (higher "x")



4. Compute $\int_{\pi/6}^{\pi/2} \int_0^{\sin\theta} 6r\cos\theta\,dr\,d\theta$.

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(3r^2 \Big|_0^{\sin\theta} \right) \cos\theta\,d\theta$$

$$= 3 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2\theta \cos\theta\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$\int u^2\,du$

$$= 3\left(\frac{\sin^3\theta}{3} \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} \right) = 1 - \left(\frac{1}{2}\right)^3 = \frac{7}{8}$$