MATH 241 Spring, 2010 Quiz #6 Name: MJ

For full credit you must show sufficient work that the method of obtaining your answer is clear. There is no need to "simplify" answers.

1. Let $z = f(x,y) = 2 + xy + 3x^2$, $\mathbf{a} = \langle 3, -4 \rangle$, and P be the point $(-1, 3)$.

   a. Compute the gradient of f.

$$\vec{\nabla} f = \langle f_x, f_y \rangle = \langle y + 6x, x \rangle$$

For (b), $\vec{\nabla} f(P) = \langle -3, -1 \rangle$.

   b. Compute the directional derivative of f at P in the direction of $\mathbf{a}$.

Need a unit vector $\hat{u} = \frac{1}{\|\vec{a}\|}\vec{a} = \frac{1}{\sqrt{9+16}}\vec{a}$

$= \langle \frac{3}{5}, -\frac{4}{5} \rangle$

$D_{\vec{u}} f(-1,3) =$

$\vec{\nabla} f(-1,3) \cdot \hat{u} = \langle -3, -1 \rangle \cdot \langle \frac{3}{5}, -\frac{4}{5} \rangle$

$= -\frac{9}{5} + \frac{4}{5} = -1$

   c. What is the maximum value among all directional derivatives of f at P?

$$\|\vec{\nabla} f(P)\| = \sqrt{9+1} = \sqrt{10}$$

   d. Give an equation for the tangent plane to the surface $z = f(x,y)$ at the point $(-1, 3, f(-1,3))$.

$x_0 = -1$, $y_0 = 3$ $z_0 =$ $f(-1,3) = 2 - 3 + 3 = 2$

Use $\vec{N} = \langle f_x(-1,3), f_y(-1,3), -1 \rangle = \langle -3, -1, -1 \rangle$

$-3(x+1) - 1(y-3) - 1(z-2) = 0$

$-3x - y - z = -2$

OR $z = L(x,y) = f(-1,3) + f_x(-1,3)(x+1) + f_y(-1,3)(y-3)$

$z = 2 - 3(x+1) - 1(y-3)$

$= 2 - 3x - y$

2. Consider $w = g(x, y, z) = x^2 z - yz^3$.

a. The point Q $(1, -1, 2)$ is on which level surface for w (or g)?

$w = 2 - (-8) = 10$

b. Determine an equation for the tangent plane to this surface at Q.

Use $\vec{N} = \vec{\nabla} w = \vec{\nabla} g$ at Q. ← ⊥ level surface

$\vec{\nabla} g = \langle g_x, g_y, g_z \rangle$

$= \langle 2xz, -z^3, x^2 - 3yz^2 \rangle$

$\vec{\nabla} g(Q) = \langle 4, -8, 1-(-3)(4) \rangle$

$= \langle 4, -8, 13 \rangle$

$4(x-1) - 8(y+1) + 13(z-2) = 0$

$4x - 8y + 13z = 4 + 8 + 26 = 38$