MATH 241 Spring, 2010 Quiz #4 Name: M^2

For full credit you must show sufficient work that the method of obtaining your answer is clear.

1. The function $f(x,y) = \frac{xy-y^2}{x^2}$ is defined everywhere on the (x,y)-plane except along the y-axis. Show that $\lim_{(x,y)\to(0,0)} f(x,y)$ fails to exist by computing the limit along the lines $y = mx$ as x approaches 0 (here m is any constant).

1

$$\lim_{x\to 0} \frac{x(mx)-(mx)^2}{x^2} = \lim_{x\to 0} \frac{(m-m^2)x^2}{x^2} = \lim_{x\to 0}(m-m^2) = m-m^2$$

$\frac{1}{2}$ These values are different for different values of "m", e.g. $m=0$, $m=\frac{1}{2}$, $m=-1$

2. Compute the first partial derivatives (where the function is defined) with respect to each of the independent variables. There is no need to "simplify". Your answers should be in the form: derivative (in some notation) equals calculation.

a. $z = f(x,y) = x^3y^{-2} + e^{xy}$.

1 $\frac{\partial z}{\partial x} = f_x = 3x^2y^{-2} + e^{xy}y$

1 $\frac{\partial z}{\partial y} = f_y = -2x^3y^{-3} + e^{xy}x$

b. $w = g(r,s,t) = r\sin(rs^2) + \ln(t-s)$

2 $\frac{\partial w}{\partial r} = g_r = r\cos(rs^2)(s^2) + \sin(rs^2)$ product rule!

2 $\frac{\partial w}{\partial s} = r\cos(rs^2)(2rs) + \frac{1}{t-s}(-1) = g_s$

1 $\frac{\partial w}{\partial t} = g_t = \frac{1}{t-s}$

$\Delta x = 0.1$
$\Delta y = -0.4$

3. If $f(1,-2) = 3$, $f_x(1,-2) = 5$, and $f_y(1,-2) = 2$ compute the locally linear (or tangent plane) approximation to $f(1.1,-2.4)$.

$1\frac{1}{2}$

$$f(1.1,-2.4) \approx f(1,-2) + f_x(1,-2)\Delta x + f_y(1,-2)\Delta y$$
$$= 3 + 5(0.1) + 2(-0.4) = 2.7$$