MATH 172 Fall, 2009 Quiz #9 Name: M^2

This last quiz will be a take-home. It is due in class after Thanksgiving. For full credit you must show sufficient work to justify your answers. **You may not consult with anyone else**, but you may use notes, text, worksheet solutions, etc.

1. A predator exhibits a type II (Michaelis-Menton) functional response to prey abundance V (measured in thousands) by having a per capita kill rate $R = \dfrac{12V}{4+V}$.

   a. What is a good approximation of R if V is very small?

   In the case $4+V \approx 4$ so $R \approx \dfrac{12V}{4} = 3V$ (straight line, slope $= 3$)

   b. What is a good approximation of R if V is very large?

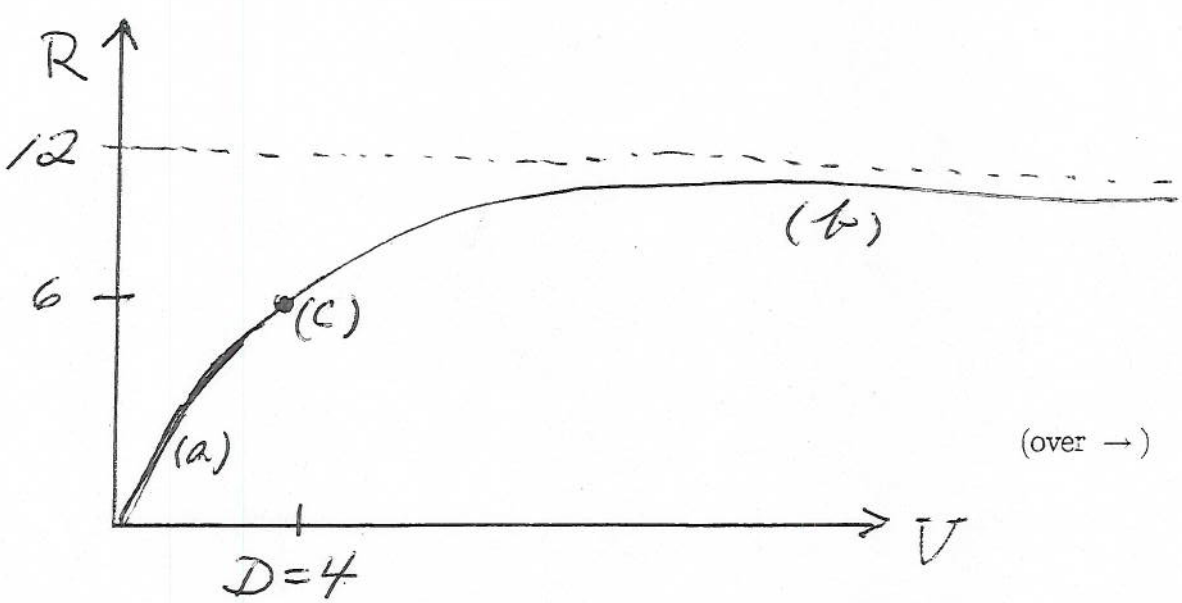
   In this case $4+V \approx V$, so

   $$R \approx \frac{12V}{V} = 12 \text{ (constant)}$$

   c. At what value of V is R one half of the maximum kill rate? What is the maximum kill rate?

   If $V=4$, then $R = \dfrac{12 \cdot 4}{8} = 6$

   d. Plot R as a function of V, exhibitng the features of a., b., c.



(over →)

2. This was a problem from your test, which we will extend just a bit here. We had the following continuous model of a predator-prey system (hares and lynxes, let's say).

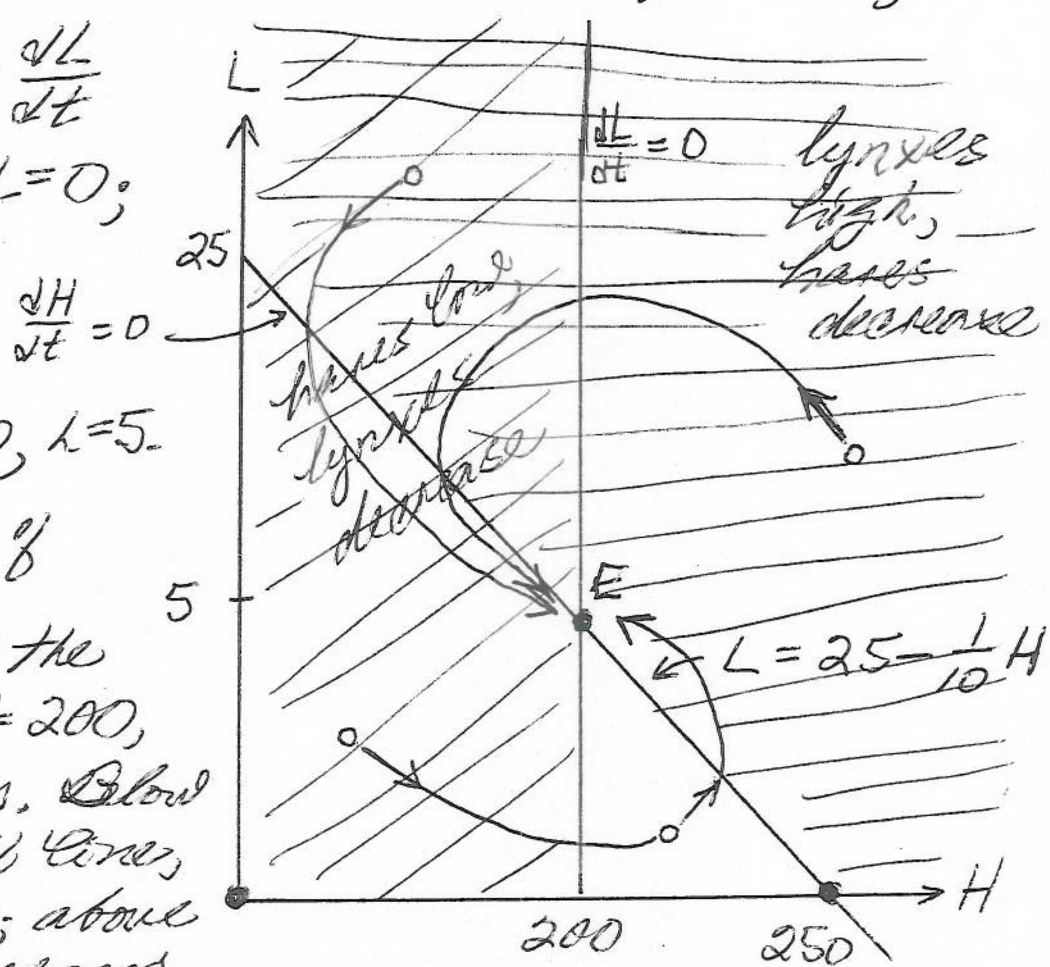
$$\frac{dH}{dt} = 0.5H(1-\frac{H}{250}) - 0.02HL = H\left(\underbrace{0.5(1-\frac{H}{250}) - 0.02L}_{\text{set}=0\text{, solve for } L:}\right)$$
$$\frac{dL}{dt} = -0.8L + 0.004HL = L(-0.8 + 0.004H)$$

$L = 25 - \frac{H}{10}$

You will recall that I asked you to mark the coordinates of the equilibrium pts (heavy dots), and place the predator population arrows (up or down), victim population arrows (left or right), and net population change arrows at the open dots. What you will do now is actuallly compute the population phase portrait trajectories. Convert these equations to discrete equations, and use Euler's method. So, for example you will have $H_{n+1} = H_n + \Delta H$ where $\Delta H \approx (\Delta t)\frac{dH}{dt}$. You may use $\Delta t = 0.25$, and of course $\frac{dH}{dt} = H_n\left(0.5(1-\frac{H_n}{250}) - 0.02L_n\right)$. You will have to do something similar for L, and then turn these into u and v for the calculator. Pick two different starting coordinates (H_0, L_0), corresponding to two of the open dots in the picture, and run n long enough so that you can determine if the equilbrium marked E is stable, unstable, or neutral. Give tables of values or graphs to illustrate your conclusion. (Use extra sheets if necessary.)

$\frac{dH}{dt} = 0 = \frac{dL}{dt}$ at $H=0, L=0$; $H=250$ $L=0$; and $H=200, L=5$.

To the left of $H=200$, L decreases; to the right of $H=200$, L increases. Below the slanted line, H increases; above it, H decreases.



$$H_{n+1} = H_n + \Delta H \approx H_n + (\Delta t)\left(\frac{dH}{dt}\right)$$

$$= H_n + (0.25)H_n\left(0.5\left(1 - \frac{H_n}{250}\right) - 0.02L_n\right)$$

$$L_{n+1} = L_n + \Delta L \approx L_n + (\Delta t)\left(\frac{dL}{dt}\right)$$

$$= L_n + (0.25)L_n(-0.8 + 0.004H_n)$$

Become

u(n)= u(n-1) + (0.25)* u(n-1) * (0.5*(1 -
u(n-1)/250) - 0.02 * v(n-1))

v(n) = v(n-1) + (0.25)* v(n-1) * (-0.8 +
0.004* u(n-1))

nMin = 0

u(nMin) = 80 or 220 or 300 or 100, etc.

v(nMin) = 3 or 2 or 7 or 30, etc.

n	0	5	10	15	20	25	30	35	40
H_n = u(n-1)	80	108.8	141.6	170.9	194.0	209.9	219.6	224.8	227.2
L_n = v(n-1)	3	2.46	2.07	1.84	1.74	1.75	1.82	1.93	2.06

45	50	55	60	65	70	75	80	85	90
227.9	227.7	226.9	225.9	224.8	223.6	222.4	221.2	220.1	219.1
2.20	2.34	2.47	2.61	2.73	2.85	2.97	3.08	3.18	3.28

95	100	105	110	115	120	125	130	135
218.1	217.1	216.2	215.3	214.5	213.7	213.0	212.3	211.6
3.38	3.46	3.55	3.63	3.70	3.77	3.84	3.90	3.96

140	145	150	155	160	165	170	175	200
211.0	210.4	209.8	209.3	208.8	208.3	207.8	207.4	205.6
4.02	4.07	4.12	4.17	4.22	4.26	4.30	4.34	4.50

n=180 corresponds to t = 45

In 2nd TblSet, I used ΔTable = 5; no steps are skipped in the calculator (you have to wait!) but just every 5th one is displayed. I kept resetting TblStart to pick off where the last screen left off. Of course you could "ask" for u, v values for specific n values. Once you get some idea of the uv window you'll need, you can just ask to go up to $n=250$ or 300 or 400 and come back later when the graph appears. 2nd ▶ with TRACE enables you to pick up u, v values along the graph, or you can enter specific values of n.

In the end you should find that the equil $H=200$, $P=5$ is stable, and that no matter where you start the trajectories of $(H(t), P(t))$ spiral in pretty directly (see illustration).

Here's a continuation of the table above ($H=80$, $P=3$ at $n=0$), now using ΔTbl = 25:

n	225	250	275	300	325	350	400
u_n	204.2	203.2	202.4	201.8	201.4	201.0	200.6
v_n	4.62	4.72	4.79	4.84	4.88	4.91	4.95