MATH 172 Fall, 2009 Quiz #6 Name:____________________

For full credit you must show sufficient work to justify your answer. You may use the following: (1) an affine continuous model $\frac{dQ}{dt} = aQ + b$ has an explicit solution $Q(t) = Ce^{at} + Q^*$, where Q^* is the equilibrium value, and C can be determined from the initial condition; (2) an affine discrete model $Q_{n+1} = aQ_n + b$ has an explicit solution $Q_n = Ca^n + Q^*$, where Q^* is the equilibrium value, and C can be determined from the initial condition.

1. a. Formulate an affine discrete model in which 80% of drug in the bloodstream from one day to the next is used up, but the remainder is reinforced by a maintenance dose of 40 mg/day. Let A_n denote the amount of drug present in the bloodstream on day n.

   b. What is the equilibrium (steady state) amount of the drug in the bloodstream?

   c. Give an explicit formula for A_n if the initial dose is $A_0 = 10$ mg.

   c. Compute A_1 , A_2 , A_{10} , A_{20} either from the formula of part (c) or from the model of part (a) with the help of your calculator.

(over $\rightarrow$)

d. Describe the long term behavior of A_n ; does it increase, decrease, oscillate, tend towards or away from the equilibrium? Does the equilibrium value you found in part (b) appear to be stable or unstable? Explain verbally and / or graphically.

2. In a good, but shrinking, wooded habitat, a population of snakes $S = S(t)$ growing at a **per capita rate** of 3% yr $^{-1}$, but 54 migrate away over the course of each year. Write an affine continuous model equation for this situation, and solve it, assuming that the initial snake population is 1,600. What exactly happens to the snake population in the long term, and how do you know? Bonus: if the population is growing, compute the doubling time; if the population is shrinking, compute the extinction time.