MATH 172 Fall, 2011 Quiz #2 Name: $M^2$

3

1. ~~Give the~~ updating equation (also known as the recurrence equation) for the length $\ell_n$ ~~of a chain of $n$~~ grocery buggies, where each buggy is 3.5 feet long, and when you push a new buggy into the chain, only 6 inches sticks out. Note that the pattern doesn't really begin until you actually have a buggy, so $\ell_0$ is not defined, $\ell_1 = 3.5$, and $\ell_2 =$ 4.0, $\ell_3 =$ 4.5. Then give the solution equation for $\ell_n$ in terms of $n$. $6'' = 0.5$ ft

| $n$ | $\ell_n$ | $\Delta\ell$ |
|---|---|---|
| 0 | | |
| 1 | 3.5 | 0.5 |
| 2 | 4.0 | 0.5 |
| 3 | 4.5 | 0.5 |
| 4 | 5.0 | 0.5 |
| ⋮ | | |
| $n$ | | |

$\frac{1}{2}$ updating: $\ell_{n+1} = \ell_n + 0.5$

1

$$\ell_n = 3.5 + (n-1)(\Delta\ell)$$
$$= 3.5 + (n-1)(0.5)$$
$$= 3.0 + 0.5n$$

$1\frac{1}{2}$

or there is a fictitious $\ell_0 = 3.0$, and

explicit solution $\ell_n = 3.0 + 0.5n$ fits the standard pattern

2

2. During the 1980's Costa Rica had the highest deforestation rate in the world at 2.9% per year. Deforestation (meaning loss of forested land) is a continuous process.

a. If $F(t)$ is the amount of forested land, write the model equation for this process.

1

$$F'(t) = -0.029F(t)$$

$\uparrow$ loss of $F(t)$ $\quad r = -0.029$

b. Give the explicit solution to this equation.

1

$$F(t) = F_0 e^{-0.029t}$$

(over $\rightarrow$)

3. Suppose a population $B(t)$ of bacteria is growing over time so that the **per capita** rate of increase is 0.2% per hour. At the same time 4 mg of the bacteria are withdrawn per hour. Assume that this process take place continuously. (There is a vessel designed for this purpose called a chemostat.)

a. Write the model equation that describes this situation.

1

$$\underbrace{B'(t)}_{\text{net rate}} = \underbrace{0.002\,B(t)}_{\text{gain rate}} - \underbrace{4}_{\text{loss rate}} \quad \text{mg/hr}$$

b. Is there a steady state or equilibrium value for the amount of bacteria? If so, compute it.

1

Set $B' = 0$, solve for $B$:

$0 = 0.002B - 4$

$4 = 0.002B$

$B^* = 2000$ mg

* indicates equilibrium

c. What will happen to an initial population of ~~100~~ 1000 mg?

1

At $B = 1000$ mg, $B' = -2 < 0$

Since $B' < 0$, $B(t)$ will decrease.

Then $B'(t)$ will become even more negative, so $B(t)$ will decrease even faster. Eventually $B(t)$ will crash to 0.

Bonus +1