An Interdisciplinary Approach to Differential Equations for Engineers

A

Final Report

for the

USC Lilly Teaching Fellows Program 1995–6

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1 Background

In an ongoing effort to improve the quality of its undergraduate instruction, the Department of Mathematics at the University of South Carolina has begun a systematic review of its undergraduate curriculum. The initial efforts have focused on the Science and Engineering Calculus sequence (MATH 141 and 142). Revisions include reducing class size, increasing contact hours with faculty, updating the syllabi, and incorporating technology, group projects and enrichment workshops into the courses. The mathematics course most frequently taken upon completion of the Calculus sequence is Elementary Differential Equations, MATH 242. This is also often the last required mathematics course in the Engineering curriculum.

My research frequently deals with the mathematical and numerical analysis of differential equations that model physical phenomena. I am disappointed when students talk about differential equations as a course whose main purpose is to memorize and master a large set of long, complicated, and disjoint solution techniques for differential equations. While these skills are important, I believe this completely misses the objective of the course, not to mention the essence of the subject. The "language" of differential equations provides a natural setting for discussions of such fundamental physical concepts as Newton's Laws of Motion, multi-compound chemical reactions, Maxwell's Laws for Electromagnetics, pattern formation in biology, and the Laws of Thermodynamics. I believe a major objective of MATH 242 should be the creation of a mathematical foundation that will be utilized throughout the Engineering and Science curricula.

The fact that a majority of the students enrolled in MATH 242 are majors in the College of Engineering suggested that modifications to this course should be a collaborative process involving both the Department of Mathematics and College of Engineering. For this reason I looked for a mentor from the College of Engineering and was pleased that Professor Ralph E. White, Chair, Department of Chemical Engineering agreed to play this role.

The original project proposal outlined three specific and interrelated objectives:

- to include realistic and tractable illustrations of the application of differential equations;
- to emphasize a unified (quantitative/qualitative/numeric) approach to mathematical analysis;
- to utilize group activities, including projects, to enhance the learning experience.

2 Implementation

Inclusion of Applications

Discussions of realistic applications of differential equations are often very time consuming. This fact tends to limit the interdisciplinary aspects of many introductory courses in differential equations. A complete discussion would begin with an explanation of the physical origins of the problem, including the formulation of the mathematical model and underlying assumptions and limitations. Next, the "solution" must be found. This process is complicated by the fact that many problems do not have an exact solution that can be expressed in in closed form. Thus, methods for finding approximate solutions must be discussed. The final step is to extract information from the solution to answer the original questions. It is difficult to find good applied problems that can be posed, formulated and solved within the constraints of the 50-minute lecture.

The increased emphasis on applications prompted a search for a new textbook. However, since the other sections of MATH 242 would continue to use the existing text, compatibility had to be a consideration. Additional factors in the search included early introduction of graphical and numerical solution techniques and problem sets with good selections of standard and applied problems. After consideration of approximately a dozen different texts, Professor White and I selected the book *Fundamentals of Differential Equations*, by Nagel and Saff.

To diversify and reinforce the interdisciplinary nature of the course, three guest lecturers from other universities were invited to make presentations to the class.[1] Guest lecturers for the Spring 1994 semester were Professors Robert Lopez (Rose–Hulman Institute of Technology; Mathematics), Steve Dunbar (University of Nebraska – Lincoln; Mathematics), and James Herod (Georgia Institute of Technology; Mathematics). While all three are mathematicians, each came from a different type of academic institution, and exhibited a different perspective of the balance between the theory and application of differential equations. Professor Herod led an investigation of some new models for the spread of infectious diseases; Professor Dunbar utilized an example from football to illustrate the application of differential equations to mechanics and design; Professor Lopez' lecture emphasized, in a general setting, the complementary roles of quantitative and qualitative analysis in engineering.

This phase of the project was quite successful. The increased emphasis on applications was very well received. A number of students remarked that this course was the first time they had understood "word problems". While student participation in the guest lectures was less than normal, attendance was excellent and follow-up feedback was positive. Next time I teach this course I hope to solicit guest lecturers from local engineers and scientists.

Unified Presentation of Mathematical Concepts

The diversity of concepts and problems encountered in MATH 242 can be imposing. One reason for the wide exposure is to illustrate the fundamental and complementary roles of qualitative, quantitative, and numerical analysis. Of these, only the quantitative analysis can reasonably be completed without the aid of a computer. The recent maturation of computer algebra systems (CAS) such as Maple and Mathematica creates new opportunities for the teaching of mathematics. The ability to use a single computing environment to analyze differential equations from several perspectives is particularly attractive. The specific choice of software was based on the facts that both Professor White and I were familiar with Maple and that USC had just completed the purchase of a site license for Maple (building upon earlier licenses purchased by Professor White and the Department of Mathematics).

While I have been using Maple as a research tool for several years, my in-class use had never advanced beyond a few in-class demonstrations. This changed when, in Fall 1994, the Department of Mathematics established its first computer *classroom*.[2]

The learning curve for a sophisticated software package such as Maple is steep. For this reason I decided to separate the initial introduction of Maple from the discussion of new mathematical content. The first week of the course was devoted to a review of critical material from prerequisite courses and to an introduction to Maple. While this consumed valuable class time, the time invested in developing this foundation paid dividends later in the course.

Initially, student response to Maple was less than encouraging. This was not too surprising. What was surprising was the number of students who expressed a very strong dislike for computers and resisted the use of technology as an educational tool. (These are students who want to be engineers or scientists!) While this became a favorite excuse whenever difficulties were encountered, the situation did improve as the semester progressed.

[1] Financial support for these visitors was received by the Provost's Instructional Development Fund, the College of Science and Mathematics, and the Department of Mathematics colloquium fund.

[2] Note that I distinguish a classroom from a laboratory.

By the end of the semester the opinions were almost unanimously supportive of this part of the course. A universal comment was that while the workload is higher than for a traditional section of MATH 242, the extra work resulted in an improved understanding of the role differential equations plays in scientific applications. Students with weaker backgrounds were generally less enthusiastic about the revised course. One plausible explanation is that, in spite of the fact that the CAS is capable of performing the algebraic manipulations, the computer is not able to decide which operations should be performed; this is still the responsibility of the user. The type of interaction required of the student makes it very difficult to hide deficiencies in their mathematical background. No major changes to this part of the course are planned.

Group Work and Projects

The ability to work effectively in a group has become increasingly important in today's scientific workplace. To emphasize this point, students were encouraged to work in groups for all class activities – except exams. This was particularly useful for the in-class computer exercises. Typically, one student would type while one or two partners would concentrate on the interpretation of results and the formulation of the next step in the problem, not to mention watching for Maple syntax errors.

Two group projects were assigned during the semester (see Appendix A.3 for the actual assignments). The first project, suggested by Professor White, was based on an application from a course offered in Chemical Engineering. The project description described the problem and provided a mathematical model. Each group was expected to conduct the necessary mathematical analysis to answer a specific list of questions. Final project reports were due near the middle of the semester.

The second project consisted of two parts, which were sandwiched around the first project. For the first part of this project each student identified, with faculty assistance, one application of differential equations from the discipline of their choice. This portion of the project report, due rather early in the semester, was a one-page description of this problem, including a list of relevant questions. Then, after completing the first project, each group selected one team member's application for in-depth analysis. This portion of the project report was due near the end of the semester.

The fact that the students were responsible for formulating relevant questions and providing the analysis needed to answer these questions was new to most students. While most enjoyed the opportunity to customize their project, this freedom also caused a few problems. In hindsight my expectations were probably a little too advanced for these students. In the future I will probably assemble a collection of project outlines from a wide range of applications. Each group would then select one of these projects, complete the project description and perform the necessary analysis.

The formation of groups is an issue that requires further thought. In spite of the fact that students were free to select their own groups, I received many comments about incompatible schedules and/or personality conflicts between group members. I am still searching for an appropriate balance between faculty control and student freedom in the selection of groups.

3 Evaluation

Evaluation of Student Performance

It is an unfortunate reality that nothing attracts student attention better than knowing that a topic will appear on an exam. This, combined with the students' knowledge that they will not find a complete applied problem on an exam or quiz, makes it difficult — but not impossible — to use the traditional grading devices (homework and exams) in a manner consistent with the interdisciplinary emphasis of the course. To reinforce Maple's integrated role in the course, some homework and exam problems required the use of the computer.

I assigned a significant amount of homework – five problems from each section in the text. Problems were selected to both complement and supplement the class presentations without being repetitive (see Appendix A.4 for a full listing). Many of the supplemental problems involved applications different from the ones discussed in class. Problems were collected the class meeting following the class discussion. The tight schedule did not permit the in-class discussion of questions related to the homework. I was available to answer questions before and after class, during office hours, and by electronic mail. In addition, full solutions for all problems were available from the Reserve Desk in the Mathematics Library.

Each exam (mid-term and final) included a range of problems; routine, single-step calculations, more involved multiple-step problems, and applied problems requiring a slight extension of the class discussions and homework assignments. Problems related to applications often took the form of short-answer questions which typically required either the formulation of a differential equation to model a situation or the interpretation of a solution in the language of the original problem (copies of all exams are available in Appendix A.5). The ability to understand the technical description of a situation and a familiarity with the vocabulary of differential equations were very important for these questions.

Assigning grades to the group projects was a new experience for me. With the exception of the first part of second project, each group received the same grade on each project. In light of the problems that developed in some of the groups, it seems as though this is somewhat unfair to some students. In the future I will attempt to evaluate individual performance within the group.

On first glance, the final grades for this course looked much like those for other sections of MATH 242. Several interesting observations become apparent with closer inspection. Even though homework and projects contributed equally to the final grade, the final grades were more strongly correlation with homework grades than with project grades. Also, no student failed the class if they regularly turned in homework, completed both projects, and took all exams.

Student Evaluation of the Course

Students completed two evaluations at the end of the semester. The first evaluation was the standard evaluation used in the College of Science and Mathematics; the results from these evaluations will not be released until later in the Summer. The second evaluation was one that I created to obtain more detailed information about specific aspects of the course (see Appendix A.6). Some of the questions on this form were adapted from evaluations used by Professors Matthew Miller and Maria Girardi and from an evaluation used in the Mathematics Department at Rensselaer Polytechnic Institute.

- Applications (see Table 1)
  Student assessment of the effectiveness of this course must be put into a proper context. At this point few students have a real understanding of the skills and knowledge required to become a successful engineer or scientist. Regardless of this limitation, responses to questions #7, 8, 9, 10, 12, and 13 provide hope that knowledge acquired in MATH 242 will be applied in a wide range of situations.

- Computer Algebra (see Table 1)
  The student responses to questions about the way the computer was used in this course support many of my own observations. It is particularly encouraging to note that the time spent introducing the students to Maple at the beginning of the semester was beneficial (#1, 3, 13). It also appears as though my ability to anticipate difficulties and judge time requirements is improving (#1, 4).

Several explanations for the decreased satisfaction with group activities (#5) are possible. One that I believe has merit is that since each student had access to a terminal, each student wanted to be the one controlling the keyboard. While this shows a willingness to be involved in the class, I will have to think about ways to encourage in-class group interaction.

- Student Response to Projects and Group Assignments (see Table 2)
  Student reaction to the projects is slightly less encouraging (#1, 7). The responses to question # 2 are consistent with the previously noted problems that developed in some groups. In spite of these problems, I hold fast to my belief that group projects should have an important role in this course. The responses to questions #3, 4, 5, and 6 suggest that the students share this feeling.

Question	Fall 1994 Average	Spring 1995 Average
1.	0.92	1.21
2.	1.15	1.07
3.	0.62	0.93
4.	0.31	0.93
5.	1.38	0.79
6.	1.85	2.00
7.	1.46	1.71
8.	0.85	1.00
9.	0.08	0.93
10.	0.46	1.50
11	0.46	1.21
12.	0.69	1.43
13.	0.31	-1.07

Table 1: Summary of student responses to questions regarding applications, computers and Maple in MATH 242. Averages are obtained by assigning integer values from -2 (strongly disagree) to +2 (strongly agree) to each response.

4 Mentor Relationship

This report would not be complete without acknowledging Professor White's contributions to this project. In our initial meetings I became familiar with Engineering's view of the role of MATH 242 in the Engineering curriculum. While there was substantial agreement between my personal preferences and the major points expressed by Professor White, it was sometimes difficult to adapt these to the current course syllabus. It quickly became apparent that all changes could not be made in the context of this one-year project. Discussions between Mathematics and Engineering have identified possible modifications to the Engineering and Mathematics curricula that will further enhance the effectiveness of our courses.

The interaction between Professor White and myself has extended far beyond MATH 242. I have provided Maple assistance to a number of undergraduate and graduate students in Professor White's courses. Additional research activities are included in the next section.

Question	Fall 1994 Average	Spring 1995 Average
1.	0.77	0.65
2.	0.92	0.46
3.	0.77	1.23
4.	1.04	1.00
5.	0.31	0.77
6.	0.46	1.15
7.	0.23	0.62

Table 2: Summary of student responses to questions regarding the group activities in MATH 242. Averages are obtained by assigning integer values from -2 (strongly disagree) to +2 (strongly agree) to each response.

5 Related Scholarly Activities

My mathematical research often includes a significant amount of interdisciplinary collaboration. The Lilly experience has exposed several new research opportunities with various faculty in USC's College of Engineering. One particularly exciting development is an invitation to participate in the NSF Engineering Research Center proposal that will provide funding for the Center for Electrochemical Engineering. My research expertise in the numerical solution of partial differential equations is expected to play a role in the success of this research project.

Other scholarly activities that have resulted from this project include a journal article, "Numerical, graphical and symbolic analysis of Bernoulli equations", that has been accepted for publication in *MapleTech, The Maple Technical Newsletter*, three book reviews, selection as a group leader/mentor at the NSF-sponsored workshop, "Revitalizing the Engineering, Mathematics, and Science Curricula via Symbolic Algebra", that will be held this summer at Rose-Hulman Institute of Technology. Contributed conference presentations have also been accepted at the Lilly Conference on College and University Teaching – South, to be hosted by USC, and the Eighth Annual International Conference on Technology in Collegiate Mathematics, which will take place in Houston.

Interest in the use of computer algebra as a tool for research and teaching is spreading throughout the USC campus. Based on this interest, and with financial support from the Provost's Instructional Development Fund, the College of Science and Mathematics, and the Department of Mathematics' colloquium fund, I decided to organize the Computer Algebra Seminar. Presentations included formal lectures from Professors Lopez, Dunbar, and Herod, demonstrations of interesting aspects of Maple, and hands-on sessions designed to introduce new user's to computer algebra. Professor White, and his students, have been active participants in the seminar. Professor Matthew Miller and Robert Murphy provided valuable assistance with the administration of the seminar.

The fact that the World Wide Web (WWW) was used to distribute all course documents has spread word of this project far beyond the boundaries of USC. This information has been accessed in excess of 4,700 times through April 1995. In addition, I have had e-mail exchanges with educators throughout the United States and as far away as Norway.

The increased exposure that has resulted from participation in the Lilly project has lead to requests to serve on a number of university committees. Beginning in Fall 1995 I will be a member of the Goldwater Scholarship Selection Committee and the Undergraduate Financial Aid and Scholarship Committee.

6 Summary and Conclusion

The goal of this Lilly project was to improve the effectiveness of the the course Elementary Differential Equations (MATH 242) using a three-pronged approach emphasizing the modelling of realistic applications, the integrated use of quantitative, qualitative, and numerical analysis including the computer algebra system Maple, and the development of group interaction skills. All evidence suggests that progress towards each of the three objectives has been realized.

The response from faculty colleagues has also been generally supportive. Several Mathematics faculty and graduate students have expressed interest in including some elements of my course in future sections of their courses. The Engineering faculty who served as contacts for projects were all very supportive and encouraging. I expect the interest will gradually increase as technology becomes a more accepted tool in the college curriculum.

Teaching this course required a significantly more time than a "normal" course. While this can sometimes be disruptive to research activities, I believe the student-faculty interaction is important. It is very important for students to be pushed to identify their questions and to receive a prompt answer to their questions. Personally, I find this one-to-one contact with students enjoyable – this is where much of the real learning occurs.

A Course Documents

A.1 WWW Home Page

NOTE: The Universal Resource Locator (URL) for this page is:

`http://www.math.scarolina.edu/~meade/math242/`

To access this home page, use a WWW browser such as netscape, Mosaic, or lynx.

A.2 Syllabus

A.3 Project Assignment

A.4 Homework Assignments

A.5 Exams

A.6 Supplemental Evaluation Form

A.7 Maple Worksheets

A inclusion of each Maple worksheet created for use in this course would be too large to include in this report (approximately 100 pages). Anyone interested in these files is encouraged to either acquire a copy from the WWW home page with URL `http://www.math.scarolina.edu/~meade/math242/` or to contact me personally.

B Selected References

Paul Blanchard, *Teaching differential equations with a dynamical systems viewpoint*, electronic pre-print available by anonymous FTP from Boston University, (e-mail requests for information can be sent to: `odes@math.bu.edu`), April 1995.

Robert L. Borrelli, Courtney Coleman, William E. Boyce, *Differential Equations Laboratory Workbook*, John Wiley & Sons, 1992.

R. Kent Nagel and Edward B. Saff, *Foundations of Differential Equations*, 3rd edition, Addison-Wesley, 1993.

Newsletter for the Consortium of Ordinary Differential Equation Experiments, (available by anonymous FTP from `fmc.hmc.edu` in `/pub/CODEE`; the equivalent WWW source of this information is the URL: `http://fmc.hmc.edu/~codee/home.html`), 1992–1994.

Dennis G. Zill, *A First Course in Differential Equations*, 5th edition, PWS-Kent, 1993.

Dennis G. Zill, *Differential Equations with Computer Lab Experiments*, PWS-Kent, 1995.

C Curriculum Vitæ