

Perspectives on the Maturation of Computer Algebra in the Academic World

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Three Perspectives

- Student
- Instructor
- Researcher

but first ...

a brief history of Computer Algebra (and Maple)

Year	Maple	Me
1980	MAPLE project conceived <ul style="list-style-type: none"> • Symbolic Computation Group • goal: “efficient” CAS 	Undergraduate Student <ul style="list-style-type: none"> • BGSU (Ohio)
1981		<ul style="list-style-type: none"> • Eastman Kodak Scholar
1982	Maple 1.0, 2.0, 2.1, 2.15, 2.2 <ul style="list-style-type: none"> • written in B for Unix • used locally at Waterloo 	
1983	Maple 3.0, 3.1 <ul style="list-style-type: none"> • 1st major MAPLE paper • ≈ 50 external installations 	<ul style="list-style-type: none"> • muMATH
1984	Maple 3.2 <ul style="list-style-type: none"> • distribution by WATCOM Products, Inc. • port to IBM VM/CMS and DEC VMS • convert Reference Manual to User’s Guide 	<ul style="list-style-type: none"> • B.S. (with Honors) in Math • B.S. in Computer Science Graduate Student <ul style="list-style-type: none"> • Carnegie Mellon • Intro Numerical Analysis • MACSYMA

Computer Algebra – Conception and Birth

1967 – MACSYMA project begins at MIT

1979 – muMATH-79 released for CP/M (8080) & TRS-DOS (Z80)

1980 – (Aug) Waterloo CS wants system to run MACSYMA
– (Dec) Geddes & Gonnet produce first MAPLE system

1982 – MACSYMA renamed Maxima

1998 – Maxima source enters public domain

muMATH / DERIVE

	CPU	OS	Memory
muMATH-79	8080, Z80	CP/M, TRS-DOS	48K
muMATH-80	6502	Apple II	
muMATH-83	8088	IBM PC, XT	300K
DERIVE (1988)		MS-DOS	512K
DERIVE (1996)		Windows	

- written in various forms of LISP
- muSIMP (micro (μ) Symbolic IMPlementation)
- muLISP

Personal Anecdote — Graduate Student

Introduction to Numerical Analysis (CMU, Fall 1984)

- Homework 7, Question 1 [[PDF](#)]
- Solution Evolution

1984 [MACSYMA](#)

2005 [Maple 9.5](#)

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1985	Maple 3.3 <ul style="list-style-type: none"> • WATCOM Products, Inc. handles license/distribution 	
1986	Maple 4.0	<ul style="list-style-type: none"> • graphics: ASCII
1987	Maple 4.1, 4.2 <ul style="list-style-type: none"> • 300 installations world-wide 	<ul style="list-style-type: none"> • check calculus/ode answers
1988	Waterloo Maple Software <ul style="list-style-type: none"> • centers at Waterloo, ETH Zürich, INRIA (France) 	<ul style="list-style-type: none"> • USENET <code>sci.math.symbolic</code> • <i>Mathematica 1.0</i>
1989	Maple 4.3 <ul style="list-style-type: none"> • 20 different OS 	<ul style="list-style-type: none"> • Ph.D. in Mathematics <p>Research Assistant Professor (Purdue)</p>
1990	Maple V <ul style="list-style-type: none"> • GUI • worksheets • 3D-graphics • Unix X-windows 	<ul style="list-style-type: none"> • Maple-generated FORTRAN for FEM

Personal Anecdote — Research FEM Matrix Assembly

$$\begin{aligned}Mc &= f \\m_{ij} &= \langle u_i, u_j \rangle = \int \int_{\Omega} \nabla u_i \cdot \nabla u_j \\f_j &= (f, u_j) = \int \int_{\Omega} f u_j\end{aligned}$$

- Solution Evolution

1990 Maple 4.3 script

1994 Maple V Release 3 worksheet

2005 Maple 9.5 worksheet

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1993	Maple Share Library Summer Workshop (U Mich)	<ul style="list-style-type: none"> • Hankel functions
1994	Summer Workshop (RPI) MapleTech Maple V Release 3 <ul style="list-style-type: none"> • export to \LaTeX 	<ul style="list-style-type: none"> • Lilly Fellow • Maple Ambassador • Revitalizing ESM Curriculum via Symbolic Algebra (RHIT)
1995	<ul style="list-style-type: none"> • Maple Users Group (MUG) • custom GUIs via MathEdge 	<ul style="list-style-type: none"> • Revitalizing ESM Curriculum via Symbolic Algebra (RHIT) • Computer Algebra Seminar • orthogonal curvilinear coord • ICTCM
1996	Maple V Release 4 <ul style="list-style-type: none"> • Power Edition 	<ul style="list-style-type: none"> • shooting method • <i>Fund of DE</i>, Nagel/Saff

Personal Anecdote — Research Hankel Functions

Bessel's Equation: $x^2y''(x) + xy'(x) + (x^2 - \nu^2)y(x) = 0$

$$\begin{aligned}H_\nu^{(1)}(x) &= J_\nu(x) + iY_\nu(x) \\H_\nu^{(2)}(x) &= J_\nu(x) - iY_\nu(x)\end{aligned}$$

1993 Maple V Release 2 [script](#) for [BesselH](#)
distributed with Maple Share Library

2005 Maple 9.5 [worksheet](#) (automatically updated)
eventually included in Maple as [HankelH1](#) and [HankelH2](#)
[\[online help\]](#)

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Personal Anecdote — Instructor/Student Lilly Project

- Elementary Differential Equations, Spring 1995
[WWW]
- Guest Lectures:
 - Steve Dunbar (Nebraska) [pursuit problem]
 - Jim Herod (Georgia Tech) [delay ODE]
 - Robert Lopez (Rose-Hulman) [qualitative analysis]

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Personal Anecdote — Research/Student Shooting Method

$$\begin{aligned}\frac{dy}{dt} &= f(t, y(t)) \\ y_i(a) &= \alpha_i \quad i = 1, 2, \dots, m_1 \\ y_{j+m_1}(b) &= \beta_j \quad j = 1, 2, \dots, m_2\end{aligned}$$

1994 Pre-**Shoot** Package

[[Maple V r 3](#)]

1997 **Shoot** Package (MTN [[PDF](#)])

[[Maple V r 3](#)] [[Maple V r 4](#)] [[Maple 6](#)] [[App Center](#)] [[source \(ZIP\)](#)]

2000 Post-**Shoot** Package

[[Maple 8 \(AEM, Lopez\)](#)] [[Maple 9.5 \(ODE PowerTool, Lesson 17\)](#)]

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1998	Maple V Release 5	<ul style="list-style-type: none"> • <i>Engineer's Toolkit</i>
1999		<ul style="list-style-type: none"> • S E I R model
2000	Maple 6 Maple Application Center	<ul style="list-style-type: none"> • <i>Linear Algebra</i>, Lay • Irreducibility Tests
2001	Maple 7	Undergraduate Director <ul style="list-style-type: none"> • ODE PowerTool • <i>Thomas' Calculus</i>
2002	Maple 8 <ul style="list-style-type: none"> • maplets • MapleNet 	<ul style="list-style-type: none"> • <i>Adv Eng Math</i>, Lopez • <i>Linear Algebra</i>, JRA • <i>Calculus</i>, Johnston/Mathews
2003	Maple 9 <ul style="list-style-type: none"> • Maplesoft • MapleTA 	<ul style="list-style-type: none"> • Calculus I with Maple in Blackboard
2004	Maple 9.5	<ul style="list-style-type: none"> • Lab Materials for Calculus (including projects)

Personal Anecdote — Research/Instructor/Student Parachute Problem

$$mx'' = -mg - k(t)x', \quad x(0) = x_0, \quad x'(0) = v_0$$

$$k(t) = \begin{cases} k_1 & t < t_d \\ k_2 & t \geq t_d \end{cases} \quad k(t) = \begin{cases} k_1 & t < t_d \\ k_d & t_d \leq t < t_d + \tau_d \\ k_2 & t \geq t_d + \tau_d \end{cases}$$

1997 Modelling with an Impact (MTN [[PDF](#)])

1998 ODE Models for the Parachute Problem (SIAM Review [[PDF](#)])

1999 Differential Equations for the New Millenium
(IJEE, w/Allan Struthers [[PDF](#)] [[html](#)])

[[Maple V r 3](#)] [[Maple V r 4](#)] [[Maple V r 5.1](#)] [[Maple 9.5](#)]

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Personal Anecdote — Instructor/Student Maple V for Engineers

- Addison-Wesley's Engineer's Toolkit Series
co-authored with Etan Bourkoff (EECE)
- Chapter 6: Advanced Engineering Applications
[\[PDF\]](#)
- Chapter 7: Intro to Maple Programming
[\[PDF\]](#)
- Instructor's Guide
[\[Maple V Release 4\]](#)

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Personal Anecdote — Instructor/Student `laylinalg` Package*

- Student Study Guide [[PDF](#)]
- Instructor Maple Manual [[PDF](#)]
- Data for Exercises
[[Maple 8](#)] [[Maple 9.5](#)]
- Application Projects
[[Error Correcting Codes](#)]
- Case Studies
[[Spotted Owl](#)]

**Introduction to Linear Algebra*, David C. Lay, 3rd ed., 2003.
[[Instructor Resources](#)] [[Supplements](#)] [[Technology](#)]

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Personal Anecdote — Research Irreducibility Tests for 0-1 Polynomials

2000 [cgi](#)-based Web forms
[\[Irreduc\]](#) [\[Cyclotomic\]](#)

2001 Journal of Algorithms (in press) Science Direct [\[PDF\]](#)

2004 maplet
[\[Irreduc.maplet\]](#) [\[via MapleNet\]](#)

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Personal Anecdote — Instructor/Student ODE PowerTool

- Maple Application Center [[WWW](#)]
- PowerTool [[WWW](#)]
- ODE Powertool [[WWW](#)]
- Lesson 6: Bifurcations
[[Maple 8](#)] [[Maple 9.5](#)]

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Personal Anecdote — Instructor/Student Calculus I with Maple in Blackboard

- Blackboard-based self-contained course
[Home] [Unit] [Lessons] [Homework / Quizzes / Exams]
- MapleNet / maplet
[EpsilonDelta] [Calculus1 maplets]
- MapleTA
Practice: [Q1] [Q2] [Q3] [Q4] [Q5]

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Personal Anecdote — Instructor/Student Lab Materials, Projects, and Maplets for Calculus

2002-3 pilot lecture

2003-4 test lectures

- Fall 2003 [Course] [Labs] [Week 1] [Lab 1]
- Spring 2004 [Course] [Labs] [Week 1] [Week 2] [Lab 1]

2004-5 full implementation

- Fall 2004 [Course] [Lab Assignments] [Maplets]
- Spring 2005 [Course] [Lab Assignments] [Maplets]

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The Next 25 Years

- Education
 - expand throughout curriculum
 - “natural” interface
- Research
 - specialized toolboxes
 - additional graphics
- Non-Academic
 - inter-application communication
 - increased efficiency