MAPLE AND ITS ROLE IN THE DEVELOPMENT OF A MATHEMATICIAN

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Abstract. My first uses of a CAS — muMath and MACSYMA — were far from successful. Still, I was captivated by the possibilities available with this type of software tool. In the ensuing twenty years, the software, and my uses of it, have become much deeper and very broad. This historical review of the past twenty years will highlight some of the major developments of Maple and their impact on my development as an academic and research mathematician. While my personal experiences form the overall framework for this paper, the conference presentation will emphasize Maple and the mathematics — including arithmetic, calculus, differential equations, linear algebra, and number theory.

1. Introduction. [1]

My purpose in this presentation is to provide a review of some of the highlights in the development of computer algebra systems, particularly Maple, over the past twenty years. My personal perspective on Maple's development is uniquely biased by the fact that my mathematical career has progressed from being an undergraduate to completing a Ph.D. and becoming a tenured professor. The majority of the presentation will consist of brief descriptions of specific projects and experiences with computer algebra systems during the past twenty years. The examples include applications in many different areas of mathematics — calculus, differential equations, linear algebra, number theory, ...

There are many parallels between the evolution of Maple and my progress through the academic ranks. In particular, my department began to expect increased curricular involvement from me precisely as Maple and its user interface matured to the point where such academic uses became more practical. The main body of this presentation is organized by decade.

2. The 1980s. Except for the last few months of 1989, I spent all of the 1980s as a student of one sort or another. During 1980–1984, I was an undergraduate at Bowling Green State University (Ohio) and for 1984–1989 I was a graduate student at Carnegie Mellon University (Pittsburgh).

2.1. Undergraduate Student. The first version of Maple, Maple 1.0, was released in January 1982. At that time I was at the midpoint toward earning double majors in mathematics and computer science. My

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[1] The full presentation upon which this paper is based is available online at `http://www.math.sc.edu/~meade/msw2004/`. That site includes an electronic copy of this paper, the slides used in the presentation, and an extensive list of links to supporting documents.

undergraduate courses did not provide any hint at the existence of software for the manipulation of mathematical objects other than numbers. As an Eastman Scholar I had the opportunity to intern and co-op under the guidance of Drs. Peter Castro and David Spence in the mathematics section of Management Services Division (MSD) at Eastman Kodak Company in Rochester, NY. While I have long forgotten the specifics of the projects I worked on, I do remember trying to use muMATH[2] to assist with some of the algebraic manipulations and simplifications. This was my first exposure to symbolic computing and computer algebra systems.

I have always pointed to my experience with Eastman Kodak as the motivation to pursue graduate studies in mathematics. It was only during the final preparations for this presentation that I realized that Eastman Kodak is also responsible for introducing me to the world of symbolic computation and computer algebra.

2.2. Graduate Student and Teaching Assistant. One of the first courses I took as a graduate student was *Introduction to Numerical Analysis* with Professor Max Gunzburger (now at Florida State University). Broyden's Method was one of the iterative methods for solving a nonlinear system that we studied. On a homework assignment we were given a specific problem that involved a parameter and asked to find the first five iterates, show some general patterns, and comment on the convergence of the Broyden iterates for the given problem. At that time, my linear algebra skills were underdeveloped; I decided the best way to attack this problem was to simply compute the iterates. Anyone who has tried to apply Broyden's Method by hand knows how difficult this can become. I believe I found only the first two iterates by hand.

By coincidence, a few weeks earlier I had heard about MACSYMA, and thought that this was the type of problem that was ideally suited to be done by a computer. This attempt at finding the iterates came to an abrupt halt when I consumed the entire semesters allocation of CPU time and disk space. (After a few visits to departmental and network administration offices I was back in action with increased allocations.) Much of the CPU time was consumed trying to find the correct sequence of expansion and simplification commands would yield results in the desired form. The Macsyma results were a surprise to me — and Professor Gunzburger. Today, 20 years later, my notebook computer running Maple 9.5 can complete this problem with less than 10 CPU seconds.

I do not recall any further attempts to use a CAS until Mike Albert relocated from the University of Waterloo to CMU, probably in late 1986. The version of Maple available at that time, Maple 4.0, had no graphical user interface and plots were created in ASCII with different characters used to distinguish multiple curves in a plot.

[2]A brief history of muMATH and its successor, Derive, can be found at the URL `http://www.derive-europe.com/theroots.asp?history`.

In 1988 I saw a demonstration of the original version of *Mathematica*. While *Mathematica* had a fancier user interface than either MACSYMA or Maple, it was prone to frequent crashes on the Apple Macintosh and there were notable limitations on its mathematical results.

Most of my early experiences with a CAS were recreational — or an attempt for me to avoid messy and seemingly mindless algebraic manipulations in homework assignments. I did not see any real use for these tools as an aide in my pursuit of a Ph.D. in mathematics or as a tool that had any educational use.

The addition of Mike Albert and Neil Calkin (presently at Clemson University) brought first-hand expertise with Maple. I continued to putter around with the latest versions of Maple (Maple 4.0, 4.1, 4.2, and, in March 1989, Maple 4.3.) I remember trying to find just the right set of commands to use to coerce Maple into providing answers to calculus and differential equation homework problems in a form that was comparable to what I had done by hand. Graphical output was still limited to character plots and there was still no graphical user interface. These experiments uncovered many errors in Maple's integration algorithms and provided a crash course in special functions.

2.3. Research Assistant Professor. As I completed my Ph.D. in 1989 and moved to a two-year research assistant professorship in the Center for Applied Mathematics at Purdue University my use of a CAS was almost nonexistent. My interest in CASs was renewed with the release of Maple V in late 1990. The most significant development, to me, was the Maple worksheet and the graphical user interface.[3] The Maple worksheet provided a reasonable interface for truly interactive use of Maple (at least as it was known at that time) and for the sharing of ideas, results, and questions among the ever-growing community of Maple users.

One thing that struck me about the reporting of errors and the posting of general questions about Maple was the promptness of well-written and informative responses from Keith Geddes, Gaston Gonnett, George Labahn, Mike Monagan, and many others who were regular readers of the USENET group `sci.math.symbolic`. In those days I mostly read postings and posted a few questions; I never believed I had the expertise to allow me to answers questions.

Even in these early days of the software, there were visionaries — such as Robert Lopez (now employed by Maplesoft) — who saw the educational benefits that could be realized by bringing Maple into the classroom, even putting the software in the hands of the students. Personally, I saw the interface and command syntax as major obstacles that made classroom use unfeasible. During this period my uses of Maple continued to be recre-

[3]Worksheets were first introduced with Maple 4.3, in March 1989, but Maple V, released in August 1990, was the first version for which Maple worksheets were truly functional.

ational. I did use Maple to prepare for some of my courses — calculus, differential equations, finite element methods — but never saw the opportunity or utility of bringing this technology into the classroom.

3. The 1990s. The 1990s was the decade of Maple V, with biennial updates (Releases 1 – 5). I moved to the University of South Carolina (USC) in the summer of 1991. In 1997 I received tenure and was promoted to Associate Professor. During this decade my use of Maple developed into an essential tool in my mathematics research and I started to come around to the belief that students could — and, in fact, should — be exposed to mathematical software in some of their mathematics courses. By the end of this decade, the educational uses of Maple far surpassed my use of Maple for research.

3.1. Assistant Professor. I am not quite sure how this happened, but it did not take very long for me to become known as the Maple expert for the University of South Carolina. I worked with faculty and students in mathematics, the physical sciences, and engineering on a wide range of projects.

In 1993, my work on developing non-reflecting boundary conditions for wave propagation, begun in my Ph.D. thesis and continued in collaboration with EEs at Purdue and Georgia Tech, called for the use of Hankel functions. Maple V knew about Bessel functions of the first and second kind, but not the third kind, i.e., the complex-valued Hankel functions. In one of my first attempts at extending the Maple language, and with much help from Mike Monagan, I assembled a Maple implementation of the Hankel functions.

There are several points that I want to make about this experience. The encouragement that I received from the support personnel at Waterloo Maple was great. When I first contacted them about this, I did not expect to receive any significant support or guidance; I was hoping to hear that someone had already done this — or that it was not possible. Instead, I was instructed how to display the Maple code used to implement other special functions and encouraged to use these routines as the basis for my extensions. When the project was finished, I was honored to be invited to submit my work for inclusion in the Maple Share Library. Now, the Hankel functions, `HankelH1` and `HankelH2`, are built-in functions in Maple. Many others in the Maple user community have similar experiences; together, the interests and efforts made by actual Maple users have been instrumental in guiding Waterloo Maple, and now Maplesoft, in their efforts to endow Maple with the features that are of most interest to the broader user community.

The next year I was one of three young USC faculty selected to be Lilly Fellows. My project, under the mentorship of Ralph White in Chemical Engineering, was to design and teach a Maple-based section of the differential equations course for engineers. To learn more about expectations and practices associated with the use of Maple in the classroom, I attended two workshops. The Maple Summer Workshop in Ann Arbor, MI, was the first time I put faces to many of the names and e-mail addresses that I had collected.[4] This was the first time that I realized I actually knew enough about Maple to offer assistance to others in the Maple community. I could hold my own in conversations, debates, and arguments with other Maple aficionados. The next month, I attended the Revitalizing the Engineering, Science, and Mathematics Curricula via Symbolic Algebra workshop, at Rose-Hulman Institute of Technology. Among the workshop organizers, Robert Lopez was the Maple expert. I spent the week developing the first lessons for use in the course that I was going to teach in Spring 1995 as part of my Lilly project. The course website is `http://www.math.sc.edu/~meade/math242-S95/`.

The course was a mixed success. I do believe most students came away from the course with a better understanding of what a differential equation is, why a differential equation can be useful, and some techniques for analyzing differential equations without knowing an explicit (or implicit) solution. Students had a lot of difficulty with the Maple. At that time, with Maple V, Release 2, students struggled with the syntax and the user interface was not completely user-friendly. Matt Miller and I have taught this course in similar settings in subsequent — but not recent — semesters.

Other Maple-related projects arose from the Lilly experience. Several of Ralph White's research projects called for the solution of boundary value problems for ordinary differential equations. In response to the fact that Maple's `dsolve` command does not provide general-purpose methods for solving these problems, I prepared a Maple implementation of the simple shooting method.[5]

During this same time period I completed my first project with a publisher. Addison-Wesley published my Maple worksheets and projects as a supplement to *Fundamentals of Differential Equations* by Nagel and Saff (Addison-Wesley, 1996). At about this point Etan Bourkoff, Electrical Engineering, approached me about the possibility of co-authoring an introductory text that would be appropriate for ENGR 101. The dual purposes of this course are to teach students some of the basic engineering principles and to have students learn to use Maple as a tool for engineering analysis. *Engineer's Toolkit: Maple V for Engineers* was published by Addison-Wesley in 1998.[6]

[4]Remember, this was before the WWW was prevalent.

[5]Later, this work would be reformulated as the `Shoot` package. This package was updated in February 1996 for inclusion in the Maple Share Library with Maple V, Release 3. The most recent version of the `Shoot` package is for Maple 6 (`http://www.math.sc.edu/~meade/maple/Shoot/Shoot.zip`). Very few, if any, changes should be required to bring `Shoot` completely up-to-date. But, the approach to the shooting method used in Unit 17 of the ODE PowerTool (`http://www.mapleapps.com/powertools/des/des.shtml`) is likely to be more effective.

[6]The last two chapters are available only as PDF files on the WWW: Chapter 6,

The most significant and enduring spin-off from the Lilly project was a sequence of articles, some joint with Allan Struthers at Michigan Technical University, discussing the modeling of a parachute jump `http://www.siam.org/journals/sirev/40-2/31624.html`. These papers, and accompanying Maple materials, have been regularly updated and continue to be widely used. I am impressed by the fact that textbook authors whose exercises were shown to be flawed have taken notice of my results to improve their discussion of this interesting, and simple, model.

Another important Maple-based project supported my research concerning the convergence properties of different transmission conditions for non-overlapping domain decomposition methods for the Helmholtz equation (in two dimensions). The symbolic and programming language translation capabilities within Maple were utilized to generate the FORTRAN code for the inner products involved in assembling the appropriate linear system of equations. I defined the basis functions, either bilinear or biquadratic, and Maple evaluated the appropriate inner products and output the FORTRAN code for these expressions. A key advantage of this approach was the elimination of quadrature errors from the assembly process. This worksheet was updated over the years; the most recent version was created in 1996, for Maple V, Release 3.

4. Associate Professor. The examples in the previous section are only a sample of some of the ways that Maple became a central part of the teaching and research components of my career. I believe the breadth of my achievements in these areas, while somewhat nontraditional, were instrumental in my successful promotion to Associate Professor with tenure in 1997. Less than six months later, in January 1998, Maple V, Release 5, was released. This event marked a milestone in Maple's development. Within the Maple community the worksheet interface was now almost universally accepted and `.mws` files (and packages) were quickly becoming the most common modes for the exchange of Maple-based results.

Communication within the Maple community was facilitated by the Maple User's Group (MUG)[7], a public mailing list supported by Waterloo Maple and Maplesoft. MUG readers and posters were spread all around the globe. A few of the major contributors to the discussions on this list were (in alphabetical order) Preben Alsholm (Germany), Bill Bauldry, Bruce Char, Carl DeVore, Stan Houghton (United Kingdom), Robert Israel (Canada), Helmut Kahovec (Austria), Mike May, Harald Pleym (Norway), Thomas Richard (Germany), Joe Riel, Ross Taylor, Alexander Walz (Germany), and Francis J. Wright (United Kingdom). One benefit of the

Advanced Engineering Mathematics (`http://www.math.sc.edu/~meade/toolkit/ch06.pdf`) and Chapter 7, Introduction to Maple Programming (`http://www.math.sc.edu/~meade/toolkit/ch07.pdf`).

[7]Monthly archives of MUG postings are available from the Symbolic Compuation Group at the University of Waterloo; the URL is `http://www.scg.uwaterloo.ca/~maple_gr/`.

delay created by the periodic (and irregular) distribution of MUG postings was the possibility to see a collection of diverse responses to a single question. The more immediate distribution of postings to today's Yahoo! groups `maple-assist` and `maple-new`, created by Carl DeVore, provides more prompt feedback but, at least for me, has the tendency to discourage multiple responses.

My uses of Maple continued to be very broad. On the research front I used Maple to complete some factorizations associated with determining the stability of steady state solutions to an S E I R model for the spread of an infection that were simply not feasible to complete by hand.

Irreducibility testing of polynomials is one of the classic challenges in computer algebra. Robust algorithms are known, but their computational complexity can make these tests impractical for general use. The simplest approach is to test if the polynomial factors. For high-order polynomials this is essentially impossible. Michael Filaseta asked for my assistance with the implementation of some new algorithms he was developing for fast irreducibility tests for a special class of polynomials. We now have an algorithm for lacunary 0-1 polynomials. The Maple implementation of this algorithm is available, with approval from Maplesoft, on the WWW at `http://www.math.sc.edu/~filaseta/irreduc.html`. I like to think that this application was one of the motivations behind the development of maplets and MapleNet (`http://www.maplesoft.com/maplenet/`) just after the turn of the millennium.

5. The 2000s. Beginning with Maple 6, released in early 2000, new releases of Maple have been appearing almost annually; the current version is Maple 9.5, released earlier in 2004. The improvements in these versions of Maple that are immediately noticed by almost all users are a new user interface, the availability of new tools for creating customized user interfaces, and improvements in Maple's ability to interact with the WWW. Each of these improves Maple's suitability for use in the classroom.

While significant improvements have been made in almost all parts of Maple, I want to draw attention to the introduction of the `Maplets` package and the replacement of the `student` package with the `Student` package and its ever growing collection of extremely useful subpackages: `Precalculus`, `Calculus1`, `MultivariateCalculus`, and `LinearAlgebra`. There are now several new ways to interact with Maple — including some interfaces that do not require the user (student) to enter a formal Maple command. In short, there are new ways to use Maple for educational and research purposes.

5.1. Undergraduate Director. During this same period, I, too, was involved with several major projects with the goal of bringing Maple into the mathematics classroom. The shift in my emphasis towards educational uses of Maple coincided with, but was independent from, becoming Undergraduate Director for the Department of Mathematics in 2001.

There are three education-related projects from the first years of this decade that are most prominent.

One of the reasons *Introduction to Linear Algebra* (Addison-Wesley, 2003), by David Lay, is the most popular linear algebra text is that users are free to use MATLAB, Maple, or *Mathematica* without sacrificing any functionality. While the text refers only to MATLAB, the exact same commands with the exact same syntax are provided in the other systems. I prepared all of the Maple materials — data for homework problems, projects, case studies, applications, a chapter in the Study Guide, and a separate Instructor's Guide — for the Updated Second Edition (2000, Maple V, Release 5) and the Third Edition (2003, Maple 8). All materials are available from the publisher's website: `http://www.aw-bc.com/info/lay/`.

The Maple Share Library has been replaced by the Maple Applications Center (`http://www.mapleapps.com/`). This change to a WWW-based archive was made to allow continual upgrades and additions to this collection of resources. In addition to individual Maple worksheets addressing a specific topic or problem, the Applications Center has a several PowerTools (`http://www.mapleapps.com/powertools/`) for both research and education. A PowerTool is a collection of Maple worksheets centered around a common theme. The Ordinary Differential Equations PowerTool (`http://www.mapleapps.com/powertools/des/des.shtml`) is a collection of 35 Maple worksheets that provides complete coverage of all topics typically covered in an introductory course in differential equations.

The third, and most recent, project is *Calculus I with Maple in Blackboard*.[8] This is essentially a self-contained online Calculus I course that closely parallels the one currently taught at the University of South Carolina. The main content of the course is presented via Maple worksheets (or, for students who do not have access to Maple, their static HTML equivalents). A wide range of maplets are provided to enhance student interaction with the material without requiring as much knowledge of Maple syntax. (For students without local access to Maple, all maplets are available remotely via MapleNet (`http://www.maplesoft.com/maplenet/`). Practice sessions, homework assignments, quizzes, and exams are administered using MapleTA (`http://www.maplesoft.com/mapleta/`).

6. The Present — and Future. Shortly after I became Undergraduate Director, the department chair, Manfred Stoll, asked me to look into the possibility of replacing the two weekly recitation sessions with a single Maple-based computer laboratory experience for all students in first-year

[8]This course can be accessed via USC's BlackBoard system at the URL `http://blackboard.sc.edu`, click on **Login**, then **Preview**, then **Search** for the string `maple`; this course is usually at the bottom of the list. Alternatively, a copy of this course on a CD can be requested from Darren McIntyre at Maplesoft (`mailto:dmcintyre@maplesoft.com`.)

calculus. I taught a pilot section of Calculus I with this format in Fall 2002 (`http://www.math.sc.edu/~meade/math141-F02/`). For the past three semesters I have prepared the laboratory materials for trial sections of Calculus I and II taught by others. One of my projects for this summer is the final preparation of laboratory materials for all students in Calculus I in Fall 2004 and Calculus II in Spring 2005.

I do not know what the next project will be. The possibilities — and my interests — are almost endless.

7. Conclusion. Today we view the tools and techniques of twenty years ago as primitive. I believe that in another twenty years we will have similar opinions about today's symbolic computing environment. The growth model might not be exponential; if it is logistic, we are still well below the carrying capacity, and most likely still below the inflection point. But, I digress ....

I look forward to 2024 and hearing a review of the next twenty years at an event such as the Maple Summer Workshop.