Assignment #3: Direct Solvers for Linear Systems

Due: as soon as possible

Problems

1. This question is about a few of the details of the block diagonal pivoting algorithm that we did not complete in class.
   (a) Verify that inequality (4.4.12), on p. 167 of Golub and VanLoan, is correct. Be sure to state any hypotheses that are used.
   (b) Explain why it is possible to choose E so that it is nonsingular.
2. This problem is long, but should be a good concluding exercise for our discussion of direct solvers. We will be considering five different factorizations that can be used for solving a linear system:
   - LU factorization with no pivoting
   - LU factorization in Matlab
   - Cholesky factorization
   - symmetric LU factorization using Parlett-Reid algorithm
   - symmetric LU factorization using block diagonal pivoting

   The assignment is to answer the following questions for each of the factorizations listed above.

   (a) Identify the Matlab command that computes each of the above; if no Matlab command exists, develop an efficient implementation of the algorithm. Use matrix and/or block operations whenever possible.
   (b) Count the number of floating point operations required to complete each of the different factorizations. Attempt to have at least two terms of count (e.g., $2n^3/3 + 8n^2$).
   (c) Based on the counts obtained in (b), decide when each algorithm would be most efficient.
   (d) Generate a set of (at least) 10 different linear systems that can be used as input to the commands identified in (a). (Explain how you know these matrices meet the necessary hypotheses.)
   (e) Use the matrices obtained in (d) as input to the commands identified in (a). Compare the number of floating point operations reported by Matlab with the number of floating point operations predicted using the operation counts found in (b). (Explain any differences.)
   (f) Use the `fit` command from Maple's `stats` package to compute the best polynomial fit to the operation counts found in (e). See the on-line help (`?stats[fit]` for complete information about the `fit` command).
   (g) Compare the least-square operation counts found in (f) to the operation counts found in (b).

Additional questions, comments, and remarks:

- What pivoting scheme is used by the `lu` command? How do you know?
- All operation counts should include both operations involved in the computation of the factorization and the solution of a linear system of equations $Ax = b$.
- Explain any differences between the explicit operation count in (b) and the least-squares fit to the actual operation count in (f).
- Your completed assignment should clearly present the findings of this study. I expect a tabular format will be quite effective for several of the steps. Be creative, organized, complete, and concise.
- Please provide electronic *and* printed copies of any Matlab files and Maple worksheets (or sessions). These should be neat and adequately documented.
- I expect that this question will take considerable time to complete. Start soon, discuss your progress amongst yourselves, ask me for guidance.