Vector Differentiation Rules

1. $\nabla \cdot \nabla \times \mathbf{F} = 0$
2. $\nabla \times \nabla f = \mathbf{0}$

Spherical Coordinates

$$x = \rho \cos\theta \sin\phi \qquad x = \rho \sin\theta \sin\phi \qquad x = \rho \cos\phi$$

Change of Variables Formula

$$\int\int_D f(x,y)\, dA_{(x,y)} = \int\int_{D*} f(\mathbf{T}(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| \, dA_{(u,v)}$$

Line, Path, Surface, and Volume Integrals

1. $\displaystyle\int_C f\, ds = \int_a^b f(\mathbf{c}(t))\|\mathbf{c}'(t)\|\, dt$
2. $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{c}(t)) \cdot \mathbf{c}'(t)\, dt$
3. $\displaystyle\int\int_S \mathbf{F} \cdot d\mathbf{S} = \int\int_D \mathbf{F}(\phi(u,v)) \cdot (\mathbf{T}_u \times \mathbf{T}_v)\, dA$
4. $\displaystyle A(D) = \int\int_D 1\, dA = \frac{1}{2}\int_{\partial D} x\, dy - y\, dx$
5. $\displaystyle A(S) = \int\int_S 1\, dS = \int\int_D \|\mathbf{T}_u \times \mathbf{T}_v\|\, dA$
6. $\displaystyle\int_C P\, dx + Q\, dy = \int\int_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial x}\, dA$
7. $\displaystyle\int\int_S \nabla \times \mathbf{F} \cdot d\mathbf{S} = \int_{\partial S} \mathbf{F} \cdot d\mathbf{s}$
8. $\displaystyle\int\int\int_\Omega \nabla \cdot \mathbf{F}\, dV = \int\int_{\partial\Omega} \mathbf{F} \cdot d\mathbf{S}$

Conservative Fields

The following are equivalent:

1. for any oriented simple closed curve C, $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{s} = 0$;
2. for any two oriented simple closed curves C_1 and C_2 which have the same endpoints,
$$\int_{C_1} \mathbf{F} \cdot d\mathbf{s} = \int_{C_2} \mathbf{F} \cdot d\mathbf{s};$$
3. $\mathbf{F}$ is the gradient of some function, f (that is, $\mathbf{F} = \nabla f$);
4. $\nabla \times \mathbf{F} = 0$.