MATH 242
Prof. Meade

University of South Carolina
Spring 2000

Exam 2
March 28, 2000

Name: ______________________
SS #: ______________________

Instructions:

1. There are a total of 6 problems on 6 pages. Check that your copy of the exam has all of the problems.

2. You must **show all of your work** to receive credit for a correct answer.

3. Your answers must be written legibly in the space provided. You may use the back of a page for additional space; please indicate clearly when you do so.

Problem	Points	Score
1	27	
2	16	
3	15	
4	15	
5	12	
6	15	
Total	100	

Think Spring (or Summer)!

1. (27 points) Suppose the matrix A has been reduced to echelon form as shown below:

$$A = \begin{bmatrix} 2 & -6 & 14 & 4 & 18 \\ -1 & 6 & -19 & 4 & -6 \\ -2 & 7 & -18 & 1 & -11 \\ 3 & -8 & 17 & 3 & 18 \end{bmatrix} \sim \begin{bmatrix} 2 & -6 & 14 & 4 & 18 \\ 0 & 3 & -12 & 6 & 3 \\ 0 & 1 & -4 & 5 & 7 \\ 0 & 1 & -4 & -3 & -9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & -6 & 14 & 4 & 18 \\ 0 & 3 & -12 & 6 & -3 \\ 0 & 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & -5 & -10 \end{bmatrix} \sim \begin{bmatrix} 2 & -6 & 14 & 4 & 18 \\ 0 & 3 & -12 & 6 & -3 \\ 0 & 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Construct an LU factorization of A. (Display L and U.)

(b) Find a basis for the column space of A. (Show the entries in the vectors.)

(c) What values of k and p make the following statement true?

The null space of A is a k-dimensional subspace of R^p.

2. (16 points) Suppose the economy is divided into three sectors: manufacturing, agriculture, and service. For each unit of output, manufacturing requires 0.10 units from other companies of the sector, 0.30 units from agriculture, and 0.30 units from service. For each unit of output, agriculture uses 0.20 units of its own output, 0.60 units from manufacturing, and 0.10 units from services. For each unit of output, the services sector consumes 0.10 units of services, 0.60 units from manufacturing, but no agricultural products.

   (a) Construct the consumption matrix for this economy.

   (b) Setup the matrix equation for the production level needed to satisfy a final demand of 9 units of agriculture and 9 units of service. NOTE: *Do not solve this system of equations.*

   (c) Will the production levels found in (b) be economically feasible? Explain.

   (d) Suppose it is discovered that the service sector requires some output from agriculture, *e.g.*, the service staff needs to eat. Will the production vector be economically feasible if 0.2 units of output from agriculture are needed? What if 0.4 units of output from agriculture are required? NOTE: The answer to the questions in (c) and (d) require an explanation, but not a lot of computation.

3. (15 points) Let $A = \begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$.

(a) Is $\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$ an eigenvector of A? If yes, find the corresponding eigenvalue.

(b) Is $\lambda = -1$ an eigenvalue of A? If yes, find one eigenvector.

4. (15 points) The matrix $A = \begin{bmatrix} 3 & -3 & 1 \\ 0 & 2 & 0 \\ -2 & 6 & 0 \end{bmatrix}$ has eigenvalues $\lambda = 1$ and $\lambda = 2$.

(a) Find a basis for the eigenspace of each eigenvalue.

(b) If A is diagonalizable, construct appropriate matrices and write the matrix equation that relates these matrices with A.

5. (12 points) Mark each statement as True or False. You do not need to justify each answer.

   (a) ______ If A is an $n \times n$ matrix, then $\operatorname{rank} A + \dim \operatorname{Nul} A = m$.

   (b) ______ If A is invertible and 2 is an eigenvalue of A with eigenvector x, then -2 is an eigenvalue of A^{-1} with eigenvector x.

   (c) ______ If A contains a row of zeros, the 0 is an eigenvalue of A.

   (d) ______ A and A^T have the same characteristic polynomial.

   (e) ______ If A is diagonalizable, then A is similar to a diagonal matrix.

   (f) ______ An $n \times n$ matrix A is diagonalizable if and only if A has n distinct eigenvalues.

6. (15 points)

   (a) Suppose a 3×5 matrix A has 3 pivot columns. Is $\operatorname{Col} A = \mathrm{R}^3$? Is $\operatorname{Nul} A = \mathrm{R}^2$? Explain your answers.

   (b) Suppose $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, are vectors in R^4 and $\operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4,\} = \mathrm{R}^4$. Is the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4,\}$ a basis for R^4? Why or why not?

   (c) A 5×5 matrix A has three eigenvalues. One eigenspace is one-dimensional and one eigenspace is two-dimensional. Is it possible that A is *not* diagonalizable? Justify your answer.