Math 526 Review

Chapter 1

- Solving linear system: do elementary operations to find reduced row echelon form (RREF), and by observing the RREF, you should be able to determine some properties of the solution, such as, infinitely many, unique or no solution.
- What if you have more equations than unknowns, i.e. for an n by m matrix A, $n > m$?
- geometric view of linear systems with 2 unknowns: if unique solution, intersection of two lines; if parallel, no solution; if overlap, infinitly many solutions. Do the same consideration for 3 unknowns.
- How to fit a curve to data?

Chapter 2

- Linear combination of vectors can span a vector (sub)space.
- When you get free variables in solution of system, decompose the solution to linear combination of vectors, with letter coefficients (free variables), and then vectors give the basis for your solution space.
- Linear dependence and independence (see definition on page 96); set linear combination to be equal to 0. If the only solution is 0 the vectorss are independent, if there exist non-zero solutions, they are dependent.
- More than n vectors in $\mathbb{R}^n$ must be dependent
- "Vectors span a space" means anything in the space can be written as linear combination of the vectors. Theorem 4 on page 110 has the same meaning.

Chapter 3

- matrix-vector product should be thought of as a linear combination of column vectors of the matrix, see definition on page 114.
- "Columns of A are indep" is equivalent to "$Ax = 0$ has only solution $x = 0$".
- Matrix-matrix product, see page 143 for rules and properties ($AB \neq BA$).
- Inverse of matrix: A may be invertible if A is square; A is invertible $\Leftrightarrow$ columns indep $\Leftrightarrow$ $\det(A) \neq 0$ $\Leftrightarrow$ $Ax = 0$ has only solution $x = 0$ $\Leftrightarrow$ we can find square matrix B, such that $AB = BA = I$.
- See Theorem 5 on page 165 for properties of inverse, and other conclusions involving the inverse.

Chapter 4

- Kernel and range of transformation (or matrix), both in definition and geometic point of view.
- When you do bunch of transformations to one guy, the order does matter, which means when you do matrix-matrix multiplication, the order does matter.
- Linear transformation fixes the origin, like for matrix A, $T(0) = A0$ is always 0.
- Definition of linear transformation on page 233.

Chapter 5

- Definition of subspace: 1. 0 is in it; 2. closed under scalar multiplication; 3. closed under vector addition.
- How to prove a set W is a subspace: just check the 3 points in definition.
- $\{v_1, ..., v_p\}$ in $\mathbb{R}^n$, then Span$\{v_1, ..., v_p\}$ is a subspace of $\mathbb{R}^n$.
- A is m by n matrix, then: solution set of $Ax = 0$ (nullspace) is a subspace of $\mathbb{R}^n$, columns span a subspace of $\mathbb{R}^m$
- kernel and range of a transformation are both subspaces of $\mathbb{R}^n$ for some n.
- Definition of basis of W: 1. vectors in basis span W; 2. vectors are independent.
- Definition of dimension of W: the number of vectors in any basis.
- How to prove vectors are independent: 1. set the linear combination of them to 0; 2. try to show the only solutions of the above equation are all zeros.
- If a set of vectors span a subspace, then some subset of them is a basis for the subspace.
- If W has dimension p, then: 1. more than p vectors are definitely dependent; 2. less than p vectors won't span W; 3. in W, p vectors are independent if and only if they span W.
- Every subspace (space) has a basis.
- Suppose you know the coordinate under one basis, how to find the coordinate under another basis?(5.2.3)
- Theorem 2.3, 2.4 and Fundamental Lemma.
- Definition of column space, row space and null space of a m by n matrix A.
- Nullspace and column space of a matrix A is exactly the kernel and range of the transformation defined by A.
- Find kernel and range geometrically (page 284 4).
- How to find basis for nullspace?(by solving $Ax = 0$)
- How to find column/row space basis? (5.4.3 and 5.4.4, or you can use transpose)
- Column space and row space of a matrix A have the same dimension.
- rank = dim(column space) = dim(row space).
- if A is $m \times n$, dim(nullspace A) is $n - r$ (n-rank).
- Theorem 3.6 and 9 for square invertible matrix.
- You should generalize everything you learn before in any space.

Chapter 6

- Just be careful and double-check.
- What if you have a upper/lower-triangular matrix?
- What if you have two rows/columns the same? What if you have one row/column is a multiple of another row/column?
- An n by n matrix A: 1. exchange two rows give you negative determinant; 2. multiply one row, multiply the determinant; 3. adding a multiple of one row to another, determinant remains the same.
- Because $\det(A) = \det(A^T)$, the above properties are also true for columns.
- "Invertible" equals non-zero determinant.
- A, B both square, then $det(AB) = det(A)det(B)$;$det(A^T) = det(A)$.
- Cramer's Rule.

Chapter 7

- Eigenvector can't be 0, eigenvalue can be 0.
- To find all evalues, need to set the characteristic polynomial $det(A - \lambda I)\mathbf{x} = 0$.
- For n by n matrix, one should get characteristic polynomial of order n. Always have n evalues, but some of them can be the same.
- Consider evalue and evector geometrically.
- λ is evalue of A $\Leftrightarrow$ $A - \lambda I$ is singular, i.e. has 0 determinant.
- espace of λ is the nullspace of $A - \lambda I$.
- Know how to find geometric and algebraic multiplicty: $1 \leq$ geo mult $\leq$ alg mult.
- Know how to solve for evalues and the corresponding evectors.
- eigenbasis: if you have n distinct evalues, always can find ebasis(why?); if some evalues are repeated, it depends.(use `Evectors(A);` see if you have * columns).
- Diagonalize a matrix: One can diagonalize A if and only one can find eigenbasis for A. And put ebasis as columns to get P, evalues of A in diagonal to get K, then $P^{-1}AP = K$. How do we know P is invertible?
- Similar matrices. They have same evalues, why? (the reason is the definition of similar) and Theorem 12 on page 385.