Homework Solutions

§7.1

#1. $A = \begin{bmatrix} 2 & -1 & -1 \\ 6 & 3 & 1 \\ 2 & 1 & 5 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$

$Av = \begin{bmatrix} 4 \\ 8 \\ -16 \end{bmatrix} = 4\begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix} = 4v$ so v is an e-vector of A w/ e-value 4.

#3. (a) The matrix A has $\lambda = 1$ as an e-value with corresponding e-vector $x = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$.

(b) $\lambda = -2$ is <u>not</u> an eigenvalue of B.

(c) C has an e-value $\lambda = -4$ with geometric multiplicity 1 and e-space $\text{Span}\left\{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$.

(d) The matrix D is nonsingular (i.e., $\lambda = 0$ is not an e-value of D)

#6. $A_1 - 2A_2 + 5A_3 = 0$ means $\begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

so $\lambda = 0$ is an e-value of A (because A is singular)

and an e-vector of A with $\lambda = 0$ is $x = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$.

#7. $A = \begin{bmatrix} 8 & 6 & 2 \\ -10 & -8 & -2 \\ 5 & 3 & -1 \end{bmatrix}$.

(a) $A - 3I = \begin{bmatrix} 5 & 6 & 2 \\ -10 & -11 & -2 \\ 5 & 3 & -4 \end{bmatrix} \xrightarrow[-r_1+r_3 \to r_3]{2r_1+r_2 \to r_2} \begin{bmatrix} 5 & 6 & 2 \\ 0 & 1 & 2 \\ 0 & -3 & -6 \end{bmatrix}$

$\xrightarrow{3r_2+r_3 \to r_3} \begin{bmatrix} 5 & 6 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$5x_1 + 6x_2 + 2x_3 = 0$
$5x_1 = -6x_2 - 2x_3$

to solve $(A-3I)x = 0$ we have $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

A basis for the eigenspace of A w/ $\lambda = 3$ is $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$.

The geometric multiplicity of $\lambda = 3$ is 1.

(b) $A + 2I = \begin{bmatrix} 10 & 6 & 2 \\ -10 & -6 & -2 \\ 5 & 3 & 1 \end{bmatrix} \xrightarrow[r_1 - 2r_2 \to r_2]{r_2 + r_1 \to r_2} \begin{bmatrix} 10 & 6 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

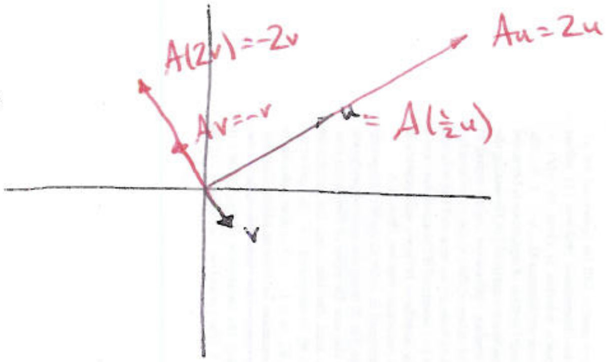
To solve $(A+2I)x = 0$ we have $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{6}{10}x_2 - \frac{2}{10}x_3 \\ x_2 \\ x_3 \end{bmatrix}$

A basis for the eigenspace of A w/ $\lambda = -2$ is $\left\{ \begin{bmatrix} -6/10 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2/10 \\ 0 \\ 1 \end{bmatrix} \right\}$.

The geometric multiplicity of $\lambda = -2$ is 2.

#8. A has 2 e-values, one e-value is positive w/ e-vector u.
The other e-value is negative, w/ e-vector v.

#9.



#17. (a) The geometric multiplicity of an e-value is the number of linearly independent eigenvectors that are associated with that eigenvalue.

(b) If λ is an eigenvalue of A, then $A-\lambda I$ is singular.

(c) If $A-\lambda I$ is singular, then λ is an eigenvalue of A.

(d) If x a nonzero vector such that $Ax=\lambda x$ then x is an eigenvector of A associated with λ.